IPNTJ Summer School on GNSS

B-1 ~ B-5
Introduction of RTKLIB
Theory of Precise Positioning (1), (2)

Tokyo Univ. of Marine Science and Technology

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2013-08-19 ~ 24 @Tokyo, Japan

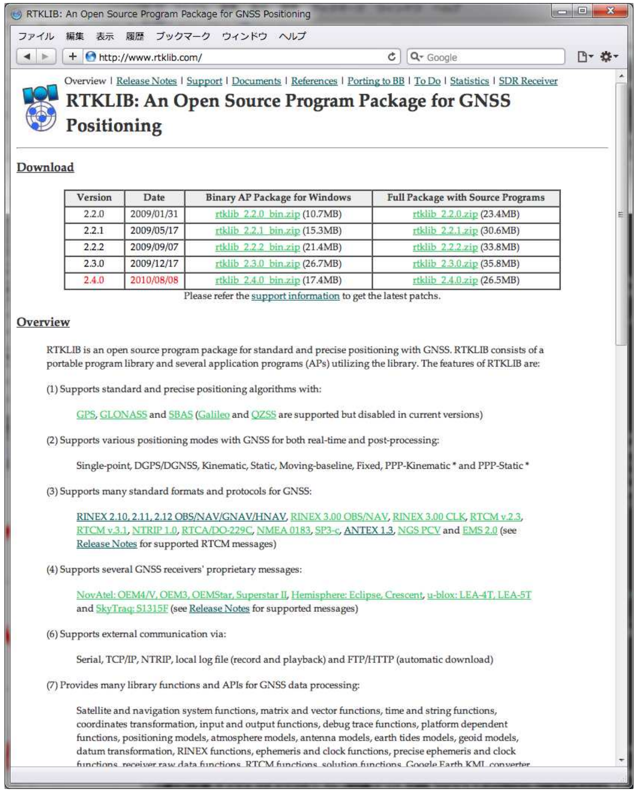
Timetable

B-1	Introduction of RTKLIB	Aug 20	8:50-10:10
B-2	Practice of RTKLIB		10:30-11:50
B-3	Theory of Precise Positioning (1)		13:00-14:20
B-4	Theory of Precise Positioning (2)		14:40-16:00
B-5	Practice of RTK		16:20-17:40
B-6	Theory of Precise Positioning (3)	Aug 21	8:50-10:10
B-7	Practice of PPP		10:30-11:50
B-8	Advanced Topics		13:00-14:20
B-9	JAXA Activities by Tateshita-san		14:40-16:00
B-10	QZSS-Demo		16:20-17:40

B-1
Introduction
of RTKLIB

RTKLIB

- **An Open Source Software Package for GNSS Positioning**
  - Has been developed since 2006
  - The latest version 2.4.2 distributed under BSD license
- **Portable APIs and Useful APs**
  - "All-in-one" package for Windows
  - CLI APs for any environments



RTKLIB: An Open Source Program Package for GNSS Positioning

ファイル 編集 表示 履歴 ブックマーク ウィンドウ ヘルプ

http://www.rtklib.com/

Overview | Release Notes | Support | Documents | References | Porting to BB | To Do | Statistics | SDR Receiver

RTKLIB: An Open Source Program Package for GNSS Positioning

Download

Version	Date	Binary AP Package for Windows	Full Package with Source Programs
2.2.0	2009/01/31	rtklib_2.2.0_bin.zip (10.7MB)	rtklib_2.2.0.zip (23.4MB)
2.2.1	2009/05/17	rtklib_2.2.1_bin.zip (15.3MB)	rtklib_2.2.1.zip (30.6MB)
2.2.2	2009/09/07	rtklib_2.2.2_bin.zip (21.4MB)	rtklib_2.2.2.zip (33.8MB)
2.3.0	2009/12/17	rtklib_2.3.0_bin.zip (26.7MB)	rtklib_2.3.0.zip (35.8MB)
2.4.0	2010/08/08	rtklib_2.4.0_bin.zip (17.4MB)	rtklib_2.4.0.zip (26.5MB)

Please refer the support information to get the latest patchs.

Overview

RTKLIB is an open source program package for standard and precise positioning with GNSS. RTKLIB consists of a portable program library and several application programs (APs) utilizing the library. The features of RTKLIB are:

(1) Supports standard and precise positioning algorithms with:

GPS, GLONASS and SBAS (Galileo and QZSS are supported but disabled in current versions)

(2) Supports various positioning modes with GNSS for both real-time and post-processing:

Single-point, DGPS/DGNSS, Kinematic, Static, Moving-baseline, Fixed, PPP-Kinematic * and PPP-Static *

(3) Supports many standard formats and protocols for GNSS:

RINEX 2.10, 2.11, 2.12 OBS/NAV/GNAV/HNAV, RINEX 3.00 OBS/NAV, RINEX 3.00 CLK, RTCM v.2.3, RTCM v.3.1, NTRIP 1.0, RTCA/DO-229C, NMEA 0183, SP3-c, ANTEX 1.3, NGS PCV and EMS 2.0 (see Release Notes for supported RTCM messages)

(4) Supports several GNSS receivers' proprietary messages:

NovAtel: OEM4/V, OEM3, OEMStar, Superstar II, Hemisphere: Eclipse, Crescent, u-blox: LEA-4T, LEA-5T and SkyTraq: S1315F (see Release Notes for supported messages)

(6) Supports external communication via:

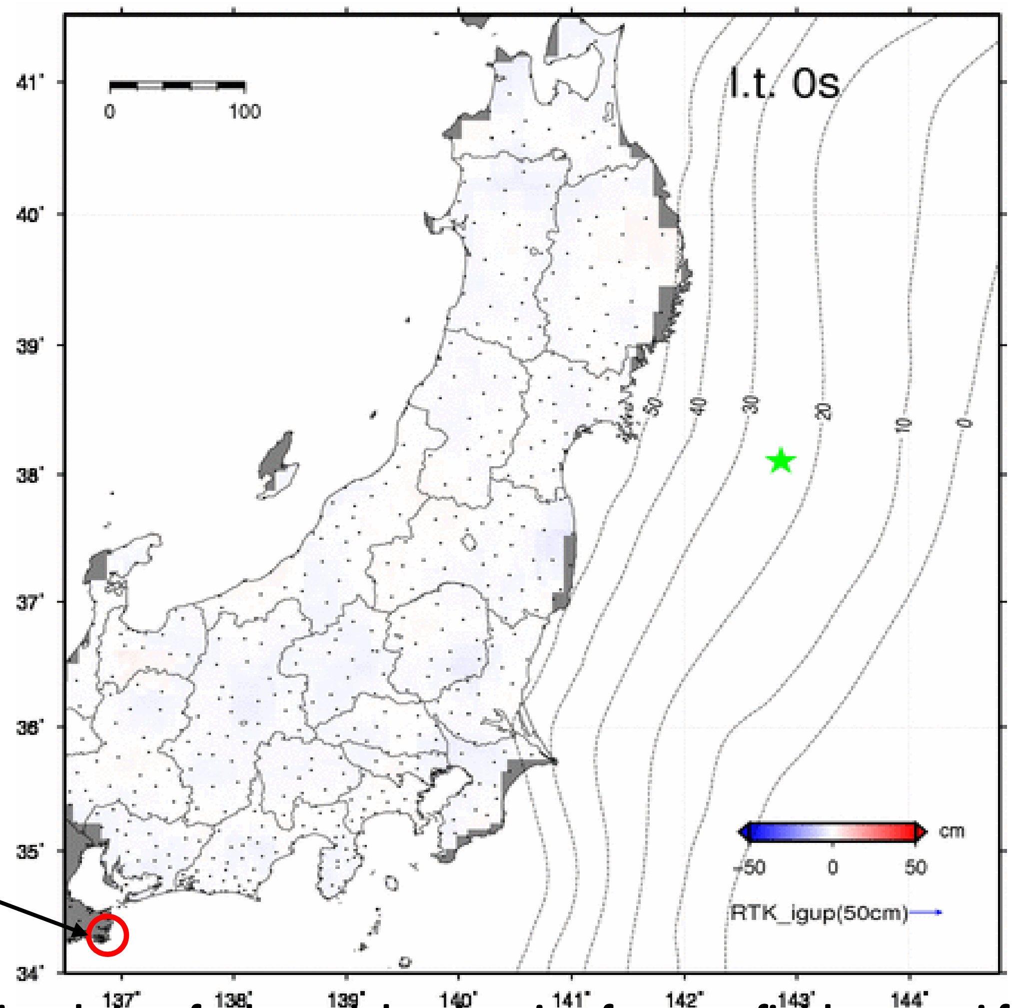
Serial, TCP/IP, NTRIP, local log file (record and playback) and FTP/HTTP (automatic download)

(7) Provides many library functions and APIs for GNSS data processing:

Satellite and navigation system functions, matrix and vector functions, time and string functions, coordinates transformation, input and output functions, debug trace functions, platform dependent functions, positioning models, atmosphere models, antenna models, earth tides models, geoid models, datum transformation, RINEX functions, ephemeris and clock functions, precise ephemeris and clock

http://www.rtklib.com or
https://github.com/tomojitakasu/RTKLIB

RTKLIB: Application



Reference Station

Y. Ohta et al., Quasi real-time fault model estimation for near-field tsunami forecasting base on RTK-GPS analysis: Application to the 2011 Tohoku-Oki earthquake (Mw 9.0), JGR-solid earth, 2012

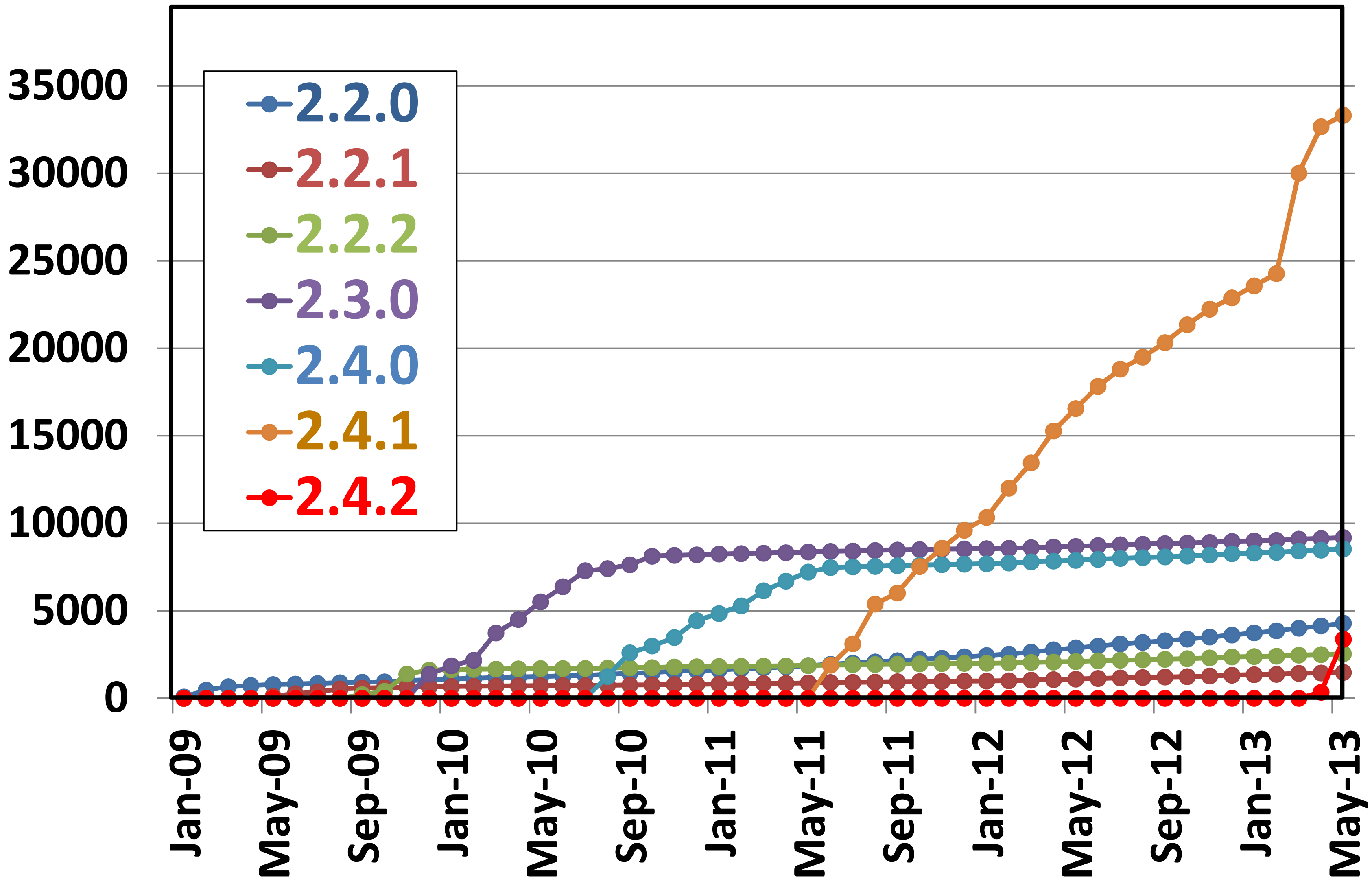
RTKLIB: History

- 2006/4 v.0.0.0 First version for RTK+C program lecture
- 2007/1 v.1.0.0 Simple post processing AP
- 2008/7 v.2.1.0 Add APs, support medium-range
- 2009/1 v.2.2.0 Add real-time AP, support NTRIP,
  start to distribute as **Open Source S/W**
- 2009/5 v.2.2.1 Support RTCM, NRTK, many receivers
- 2009/12 v.2.3.0 Support GLONASS, several receivers
- 2010/8 v.2.4.0 Support PPP Real-time/Post-processing
  PPP and Long-baseline RTK (<1000 km)
- 2011/6 v.2.4.1 Support QZSS, JAVAD receiver, ...
- 2013/4 v.2.4.2 Support Galileo, Enable BeiDou, ...

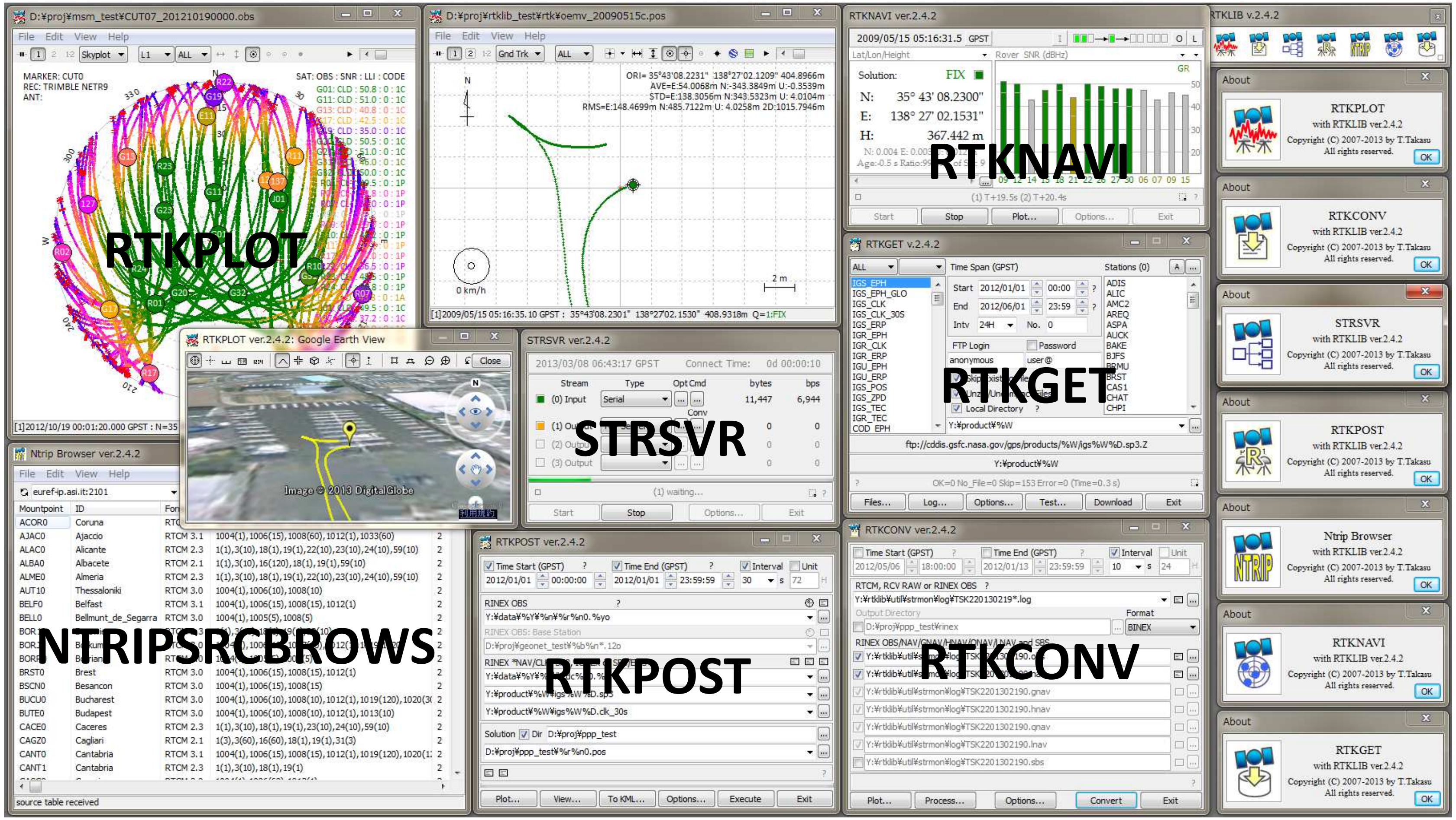
RTKLIB: Features

- **Standard and precise positioning algorithms with**:
  - GPS, GLONASS, QZSS, Galileo, BeiDou and SBAS
- **Real-time and post-processing by various modes**:
  - Single, SBAS, DGPS, RTK, Static, Moving-base and PPP
- **Supports many formats/protocols and receivers**:
  - RINEX 2/3, RTCM 2/3, BINEX, NTRIP 1.0, NMEA0183, SP3, RINEX CLK, ANTEX, NGS PCV, IONEX, RTCA-DO-229, EMS,
  - NovAtel, JAVAD, Hemisphere, u-blox, SkyTraq, NVS, …
- **Supports real-time communication via**:
  - Serial, TCP/IP, NTRIP and file streams

RTKLIB: # of Downloads

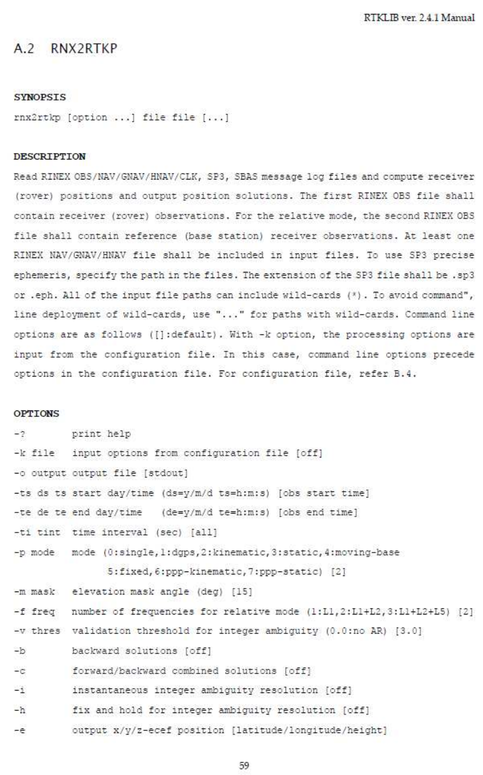


RTKLIB: GUI APs



RTKLIB: CLI APs

- **RNX2RTKP (rnx2rtkp)**
  Post-processing Positioning
- **RTKRCV (rtkrcv)**
  Real-time Positioning
- **CONVBIN (convbin)**
  RINEX Translator
- **STR2STR (str2str)**
  Stream Server
- **POS2KML (pos2kml)**
  Google Earth Converter



RTKLIB ver. 2.4.1 Manual

A.2 RNX2RTKP

SYNOPSIS

```
rnx2rtkp [option ...] file file [...]
```

DESCRIPTION

Read RINEX OBS/NAV/GNAV/HNAV/CLK, SP3, SBAS message log files and compute receiver (rover) positions and output position solutions. The first RINEX OBS file shall contain receiver (rover) observations. For the relative mode, the second RINEX OBS file shall contain reference (base station) receiver observations. At least one RINEX NAV/GNAV/HNAV file shall be included in input files. To use SP3 precise ephemeris, specify the path in the files. The extension of the SP3 file shall be .sp3 or .eph. All of the input file paths can include wild-cards (*). To avoid command", line deployment of wild-cards, use "..." for paths with wild-cards. Command line options are as follows ([]:default). With -k option, the processing options are input from the configuration file. In this case, command line options precede options in the configuration file. For configuration file, refer B.4.

OPTIONS

```
-?         print help
-k file    input options from configuration file [off]
-o output  output file [stdout]
-ts ds ts  start day/time (ds=y/m/d ts=h:m:s) [obs start time]
-te de te  end day/time   (de=y/m/d te=h:m:s) [obs end time]
-ti tint   time interval (sec) [all]
-p mode    mode (0:single,1:dgps,2:kinematic,3:static,4:moving-base
                 5:fixed,6:ppp-kinematic,7:ppp-static) [2]
-m mask    elevation mask angle (deg) [15]
-f freq    number of frequencies for relative mode (1:L1,2:L1+L2,3:L1+L2+L5) [2]
-v thres   validation threshold for integer ambiguity (0.0:no AR) [3.0]
-b         backward solutions [off]
-c         forward/backward combined solutions [off]
-i         instantaneous integer ambiguity resolution [off]
-h         fix and hold for integer ambiguity resolution [off]
-e         output x/y/z-ecef position [latitude/longitude/height]
```

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CLI Command Reference

RTKLIB: Package Structure

rtklib_2.4.2.zip

```
   /src                   : Source programs of RTKLIB libraries
        /rcv              : Source programs depending on GPS/GNSS receiv.
   /bin                   : Executable binary APs and DLLs for Windows
   /data                  : Sample data for APs
   /app                   : Build environment for APs
        /rtknavi          : RTKNAVI (GUI)
        /strsvr           : STRSVR (GUI)
        /rtkpost          : RTKPOST (GUI)
        /rtkpost_mkl      : RTKPOST_MKL (GUI)
        /rtkplot          : RTKPLOT (GUI)
        /rtkconv          : RTKCONV (GUI)
        /srctblbrows      : NTRIP source table browser (GUI)
        /rtkrcv           : RTKRCV (console)
        /rnx2rtkp         : RNX2RTKP (console)
        /pos2kml          : POS2KML (console)
        /convbin          : CONVBIN (console)
        /str2str          : STR2STR (console)
        /appcmn           : Common routines for GUI APs
        /icon             : Icon data for GUI APs
   /mkl                   : Intel MKL libraries for Borland environment
   /test                  : Test program and data
   /util                  : Utilities
   /doc                   : Document files
```

RTKLIB: APIs

```
/* matrix and vector functions */
mat(),imat(),zeros(),eye(),dot(),norm(),matcpy(),matmul(),matinv(),solve(),lsq(),filter(),smoother(),matprint(),matfprint()
/* time and string functions */
str2num(),str2time(),time2str(),epoch2time(),time2epoch(),gpst2time(),time2gpst(),timeadd(),timediff(),gpst2utc(),utc2gpst(),
timeget(),time2doy(),adjgpsweek(),tickget(),sleepms()
/* coordinates functions */
ecef2pos(),pos2ecef(),ecef2enu(),enu2ecef(),covenu(),covecef(),xyz2enu(),geoidh(),loaddatump(),tokyo2jgd(),jgd2tokyo()
/* input/output functions */
readpcv(),readpos(),sortobs(),uniqeph(),screent()
/* positioning models */
eph2pos(),geph2pos(),satpos(),satposv(),satposiode(),satazel(),geodist(),dops(),ionmodel(),ionmapf(),tropmodel(),tropmapf(),
antmodel(),csmooth()
/* single-point positioning */
pntpos(),pntvel()
/* rinex functions */
readrnx(),readrnxt(),outrnxobsh(),outrnxnavh(),outrnxnavb(),uncompress(),convrnx()
/* precise ephemeris functions */
readsp3(),readsap(),eph2posp(),satposp()
/* receiver raw data functions */
getbitu(),getbits(),crc32(),crc24q(),decode_word(),decode_frame(),init_raw(),free_raw(),input_raw(),input_rawf(),input_oem4(),
input_oem3(),input_ubx(),input_ss2(),input_cres(),input_oem4f(),input_oem3f(),input_ubxf(),input_ss2f(),input_cresf()
/* rtcm functions */
init_rtcm(),free_rtcm(),input_rtcm2(),input_rtcm3(),input_rtcm2f(),input_rtcm3f()
/* solution functions */
readsol(),readsolt(),outsolheads(),outsols(),outsolexs(),outsolhead(),outsol(),outsolex(),setsolopt(),setsolformat(),
outnmea_rmc(),outnmea_gga(),outnmea_gsa(),outnmea_gsv(),
/* SBAS functions */
sbsreadmsg(),sbsreadmsgt(),sbsoutmsg(),sbsupdatestat(),sbsdecodemsg(),sbssatpos(),sbspntpos()
/* integer least-square estimation */
lambda()
/* realtime kinematic positioning */
rtkinit(),rtkfree(),rtkpos()
/* post-processing positioning */
postpos(),postposopt(),readopts(),writeopts()
/* stream data input/output */
strinitcom(),strinit(),strlock(),strunlock(),stropen(),strclose(),strread(),strwrite(),strsync(),strstat(),strsum(), strsetopt(),
strgettime()
/* stream server functions */
strsvrinit(),strsvrstart(),strsvrstop(),strsvrstat()
/* rtk server functions */
rtksvrinit(),rtksvrstart(),rtksvrstop(),rtksvrlock(),rtksvrunlock(),rtksvrostat(),rtksvrsstat() ...
```

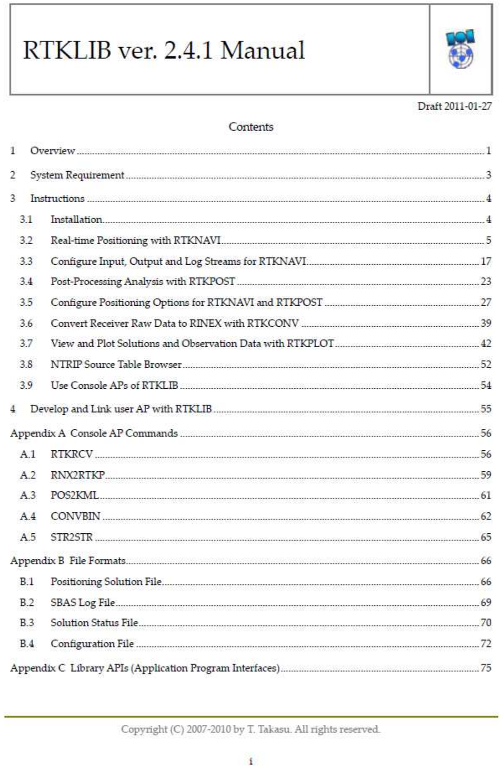
RTKLIB: Supported Receivers

Format	Data Message Types							
	GPS Raw Meas Data	GLONASS Raw Meas	GPS Ephemeris	GLONASS Ephemeris	ION/UTC Parameters	Antenna Info	SBAS Messages	Others
RTCM v.2.3	Type 18, 19	Type 18, 19	Type 17	-	-	Type 3, 22	-	Type 1, 9, 14, 16
RTCM v.3.1	Type 1002, 1004	Type 1010, 1012	Type 1019	Type 1020	-	Type 1005, 1006, 1007, 1008, 1033	-	SSR corrections
NovAtel OEM4/V, OEMStar	RANGEB, RANGECMPB	RANGEB, RANGECMPB	RAWEPHEMB	GLO-EPHEMERISB	IONUTCB	-	RAWWAAS-FRAMEB	-
NovAtel OEM3	RGEB, RGED	-	REPB	-	IONB, UTCB	-	FRMB	-
NovAtel Superstar II	ID#23	-	ID#22	-	-	-	ID#67	ID#20, #21
u-blox LEA-4T, LEA-5T	UBX RXM-RAW	-	UBX RXM-SFRB	-	UBX RXM-SFRB	-	UBX RXM-SFRB	-
Hemisphere Crescent, Eclipse	bin 96	-	bin 95	-	bin 94	-	bin 80	-
SkyTraq S1315F	msg 0xDD (221)	-	msg 0xE0 (224)	-	msg 0xE0 (224)	-	-	msg 0xDC (220)
JAVAD (GRIL/GREIS)	[R*],[r*],[*R], [*r],[P*],[p*], [*p],[D*],[*d], [E*],[*E],[F*]	[R*],[r*],[*R], [*r],[P*],[p*], [*p],[D*],[*d], [E*],[*E],[F*]	[GE],[GD], [gd]	[NE],[LD]	[IO],[UO], [GD]	-	[WD]	[~~],[::],[RD], [SI],[NN],[TC], QZSS Data, Galileo Data
Furuno GW10 II	msg 0x08	-	msg 0x24	-	msg 0x26	-	msg 0x03	msg 0x20

RTKLIB: Portability

- **Programming Language**
  - API, CUI AP : ANSI C (C89)
  - GUI AP : C++
- **Underlying Libraries**
  - TCP/IP Stack : standard socket or WINSOCK
  - Thread : POSIX (pthread) or WIN32 thread
  - GUI Widgets : Borland VCL on Windows
- **Build Environment**
  - CLI AP : GCC, MS VS, Borland C, ...
  - GUI AP : Borland Turbo C++ on Windows

RTKLIB: References



RTKLIB ver. 2.4.1 Manual

Draft 2011-01-27

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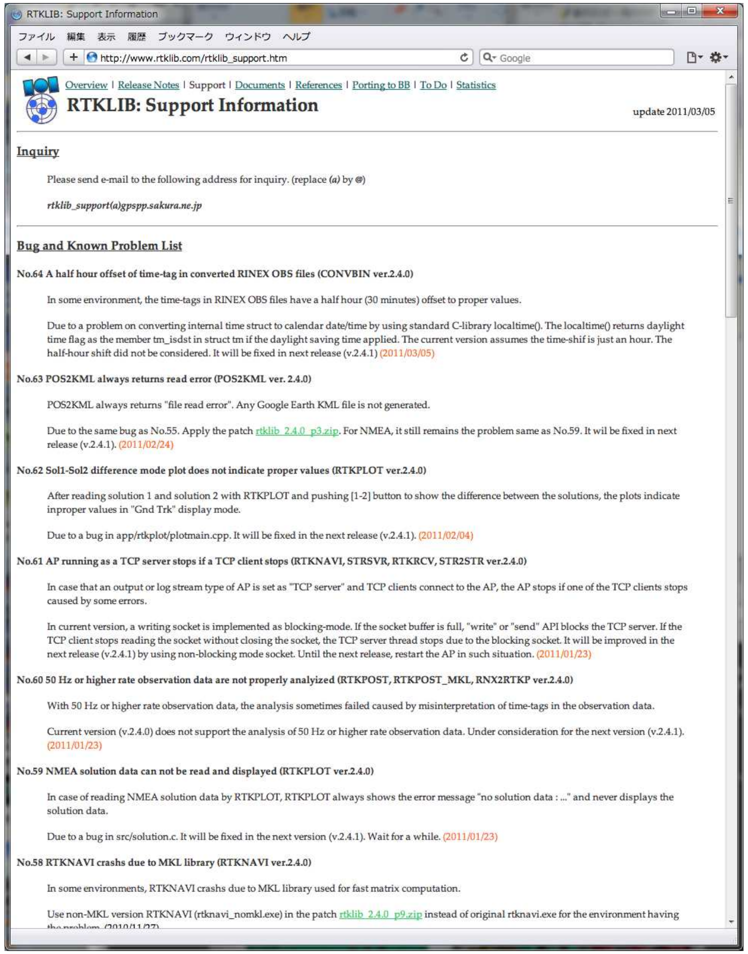
rtklib_2.4.2/doc/manual_2.4.2.pdf

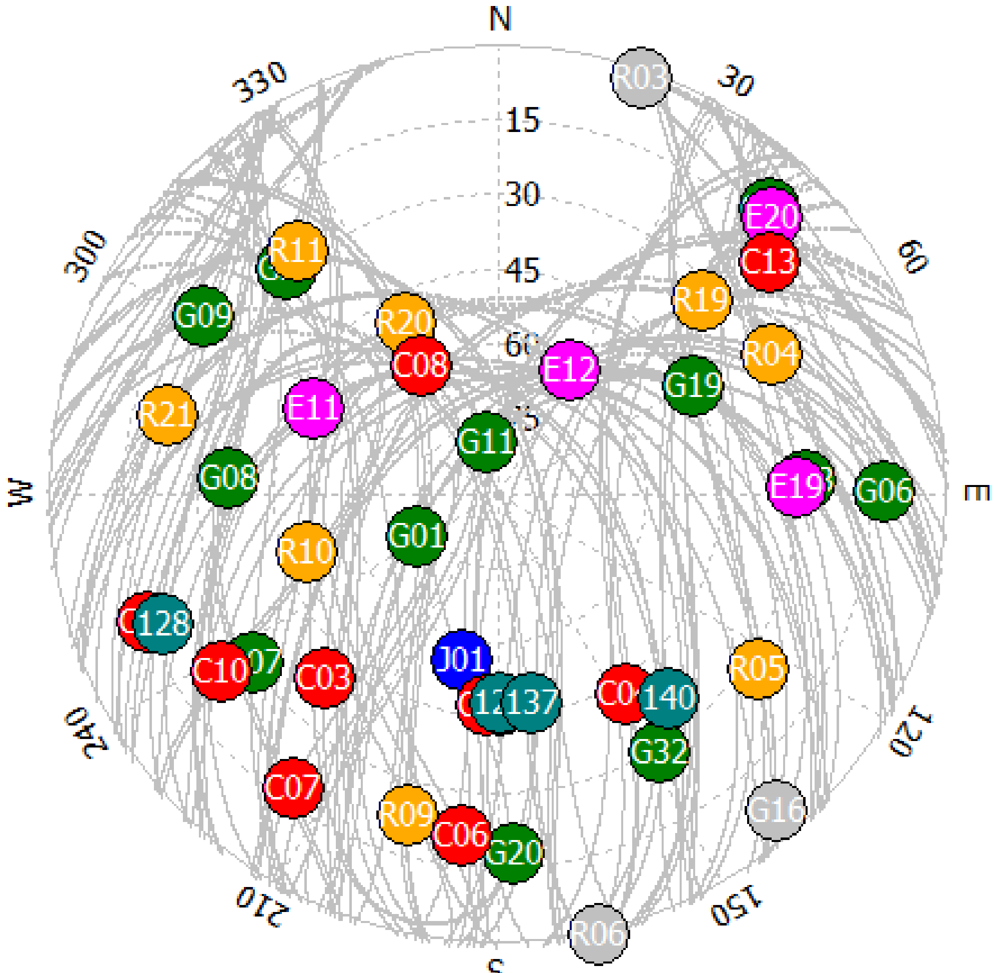


http://www.rtklib.com

RTKLIB 2.4.2: New Features

- Galileo and BeiDou supported
- Full RINEX 3 compliant, multi-signal supported
- RTCM 3.2 MSM and SSR for all GNSSs supported
- BINEX, NovAtel OEM6 and NVS supported
- Google Earth/Map View by RTKPLOT
- Satellite visibility analysis with NORAD TLE
- Data downloader AP: RTKGET added
- Data format conversion by STRSVR or STR2STR
- License Change: GPL v3 -> BSD 2-clause

RTKLIB 2.4.2: Multi-GNSS Support



2013-06-12 10:20 GPST
Visibility at Tokyo by RTKPLOT

- GPS (12)
- GLONASS (8)
- Galileo (4)
- QZSS (1)
- BeiDou (10)
- SBAS (4)

 # Total (39)
(El>10deg)

RTKLIB 2.4.2: RINEX Support

Ver.	OBS Data						NAV Messages						Met	CLK	GEO BRDC
	G	R	E	J	C	S	G	R	E	J	C	S			
2.10	O	O	O	O	O	O	N	G	N	N	-	H	-	-	-
2.11	O	O	O	O	O	O	N	G	N	N	-	H	-	-	-
2.12	O	O	O	O	O	O	N	G	N	N	-	H	-	-	-
3.00	O	O	O	O	O	O	N	N	N	N	N	N	-	C	-
3.01	O	O	O	O	O	O	N	N	N	N	N	N	-	C	-
3.02*	O	O	O	O	O	O	N	N	N	N	N	N	-	C	-

G: GPS, R: GLONASS, E: Galileo, J: QZSS, C: BeiDou, S: SBAS

* Based on draft (2012-12), O/N: RTKLIB Extension

RTKLIB 2.4.2: RTCM 3 Support

Message		GPS	GLOASS	Galileo	QZSS	BeiDou	SBAS
OBS	Compact L1	1001~	1009~	-	-	-	-
	Full L1	1002	1010	-	-	-	-
	Compact L1/2	1003~	1011~	-	-	-	-
	Full L1/2	1004	1012	-	-	-	-
Ephemeris		1019	1020	1045/6*	1044*	-	-
MSM	1	1071~	1081~	1091~	1111*~	1121*~	1101*~
	2	1072~	1082~	1092~	1112*~	1122*~	1102*~
	3	1073~	1083~	1093~	1113*~	1123*~	1103*~
	4	1074	1084	1094	1114*	1124*	1104*
	5	1075	1085	1095	1115*	1125*	1105*
	6	1076	1086	1096	1116*	1126*	1106*
	7	1077	1087	1097	1117*	1127*	1107*
SSR	Orbit Corr.	1057	1063	1240*	1246*	-	-
	Clock Corr.	1058	1064	1241*	1247*	-	-
	Code Bias	1059	1065	1242*	1248*	-	-
	Combined	1060	1066	1243*	1249*	-	-
	URA	1061	1067	1244*	1250*	-	-
	HR-Clock	1062	1068	1245*	1251*	-	-
Antenna Info		1005 1006 1007 1008 1033					

* based on draft, ~ only encode

RTK vs. PPP

	RTK	Real-Time PPP
Coverage	Local/Regional (< 1000km)	Global
Typical Accuracy	1-3 cm HRMS	2-10 cm, much depending on orbit/clock quality
Effect of Ref Movement	Hard to separate ref and user movement	Less effect by distributed ref stations
System Complexity	Simple, at least one ref station	Complicated, need many ref stations
Latency of Corrections	~ 1 s	5 ~ 25 s
Biases	Basically cancelled by DD	Need careful handling

Which is better depends on AP requirement and technology level.
RTKLIB offers both. They are user-selectable by option settings.

RTKLIB 2.4.2: PPP Models

	v.2.4.1	v.2.4.2
Satellites	GPS, GLO and QZS	GPS, GLO, QZS and **GAL**
Troposphere	Standard-Atmosphere NMF + Gradient	Standard or **GPT** NMF or **GMF** + Gradient
Ionosphere	Iono-Free LC (L1-L2)	Iono-Free LC (L1-L2**, L1-L5**) or **IONEX for single-freq**
Tidal Displacement	Solid Earth Tide: IERS 1996 Step 1 + Step 2 K1 radial only	Solid Earth Tide: **IERS DEHANTTIDEINEL.F** **Ocean Tide Loading:** **IERS 2010 with BLQ** **Pole Tide:** **IERS 2010 with IGS ERP**
Ambiguity Resolution	No (FLOAT)	**Yes with CNES Products** **(Experimental)**

B-2
Practice of RTKLIB

Preparation for Practices

- **Copy the following directory and files in the USB memory to your laptop PC.**

  school

  ¥rtklib_2.4.2

  ¥sample1

  ...

  ¥sample5

  ¥novatel

  ¥ublox

RTKLAUNCH

- **Program**
  rtklib_2.4.2¥bin¥rtklaunch.exe

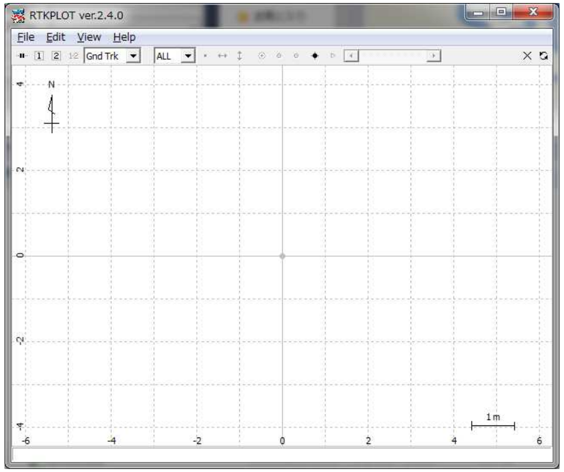
RTKPLOT STRSVR NTRIPBRS RTEGET

RTKCONV RTKPOST RTKNAVI

RTKPLOT

- **Objective**
  check visible GNSS satellites
- **Program**
  rtklib_2.4.2¥bin¥rtkplot.exe
- **Data**
  school¥sample1¥
  javad1_201102030000.obs
  javad1_201102030000.nav

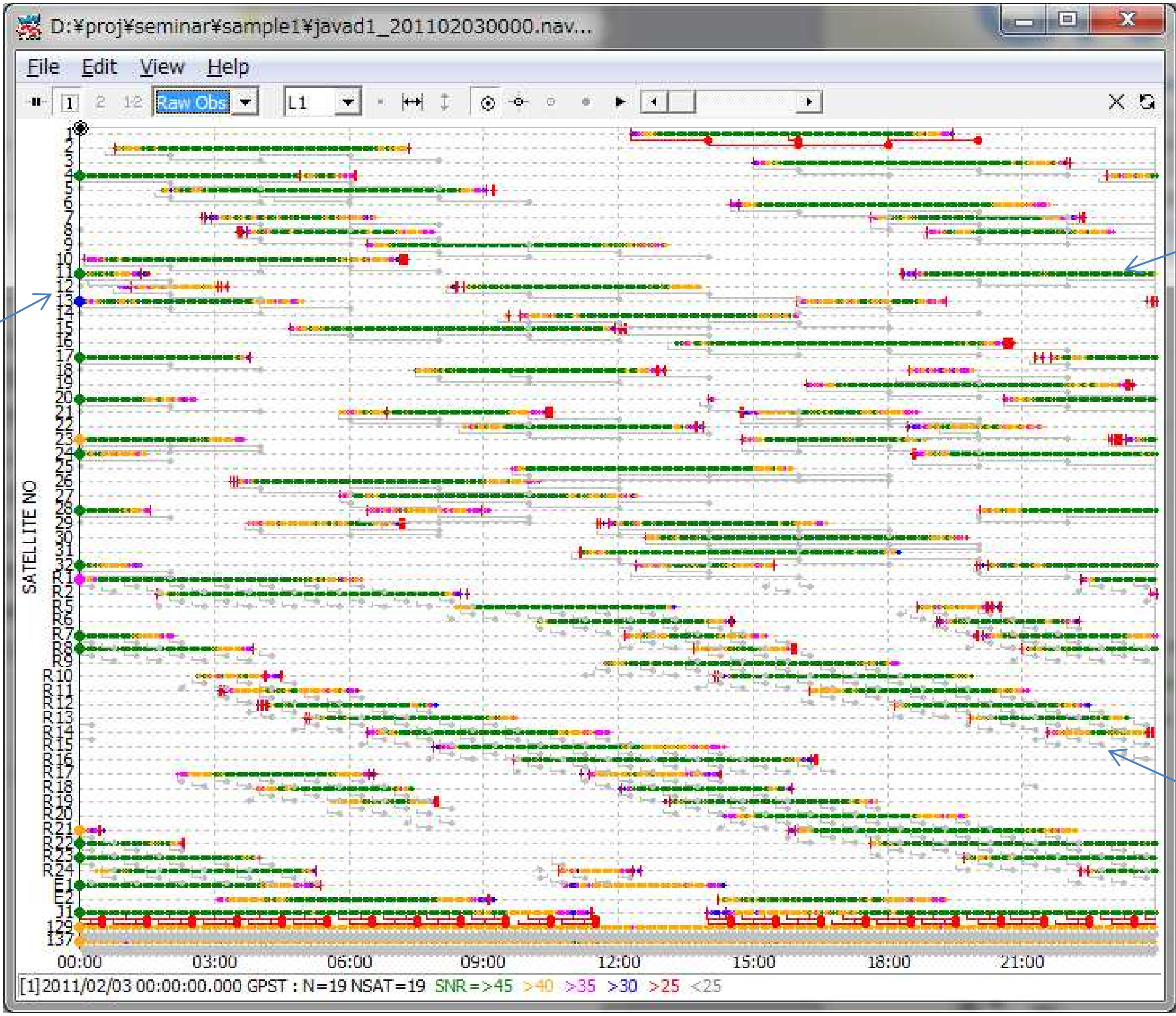
RTKLIB - RTKPLOT



JAVAD DELTA Receiver

Acknowledgment:
Sample data were captured by JAVAD DELTA receiver provided by JAXA

Satellite Visibility



Satellite ID

Gnn: GPS

Rnn: GLO

Enn: GAL

Jnn: QZS

1nn: SBAS

Tracking Data

|: Cycle-Slip

|: Parity Unknown

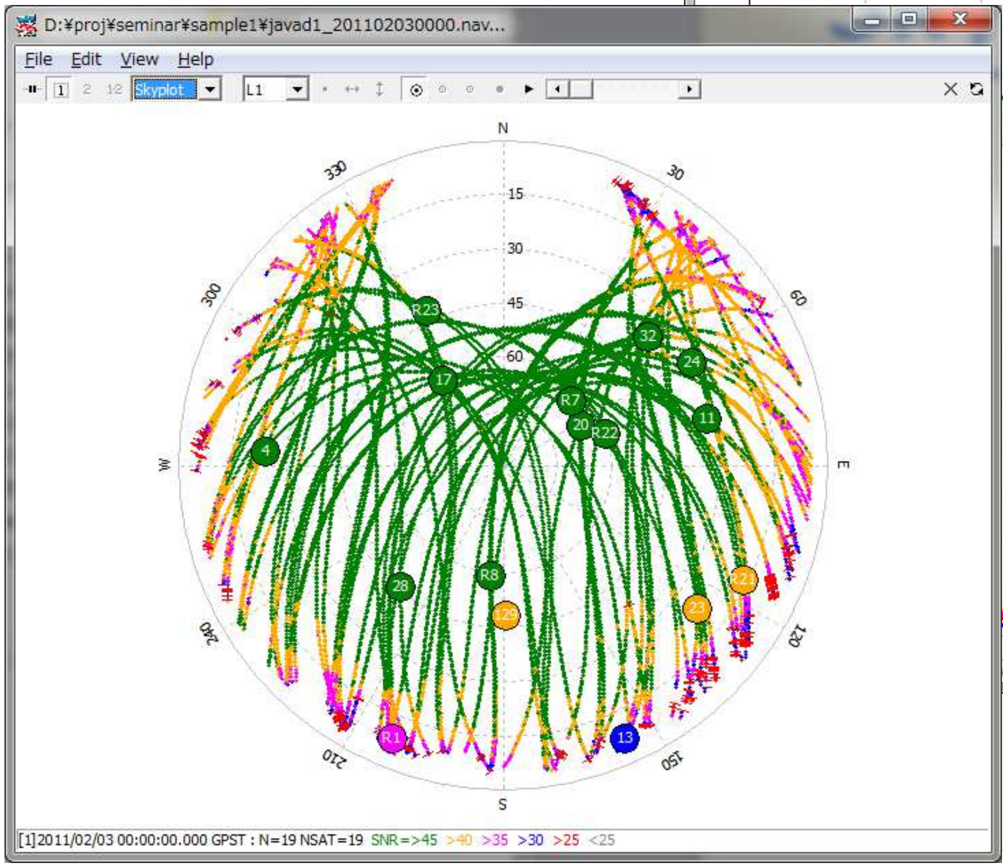
Ephemeris

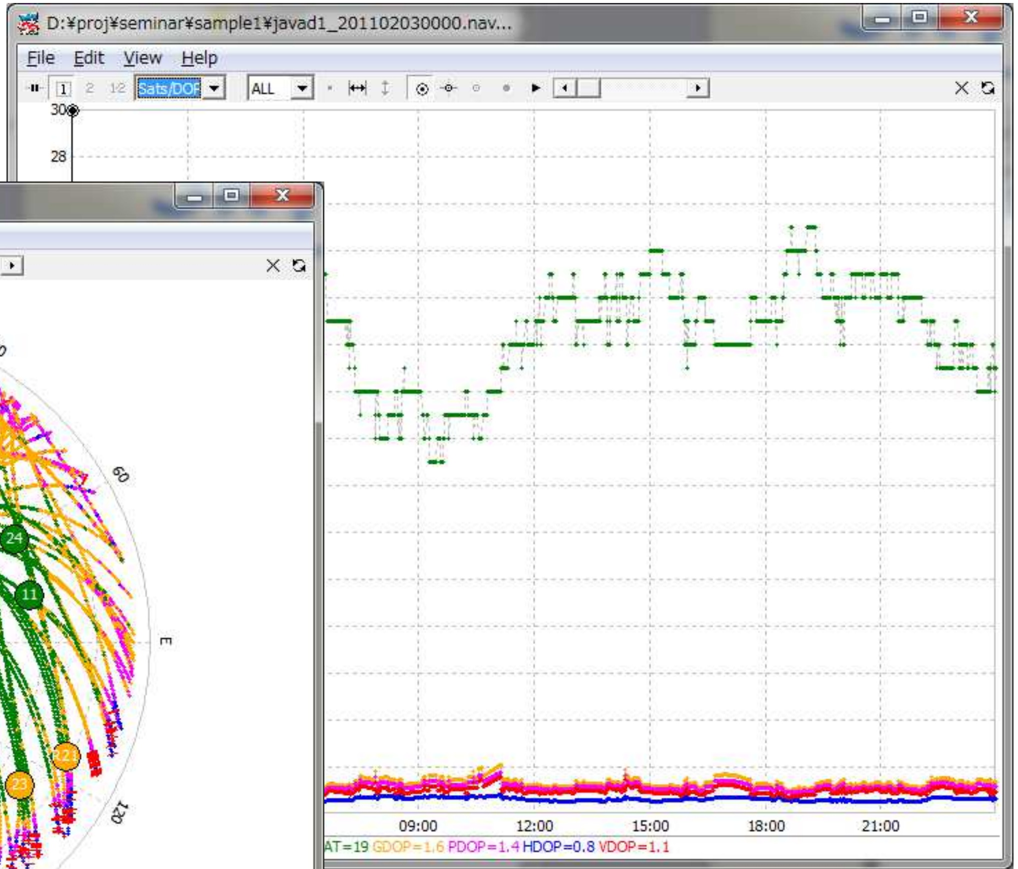
● : Toe

Red: unhealthy

Skyplot or # of Sats/DOP

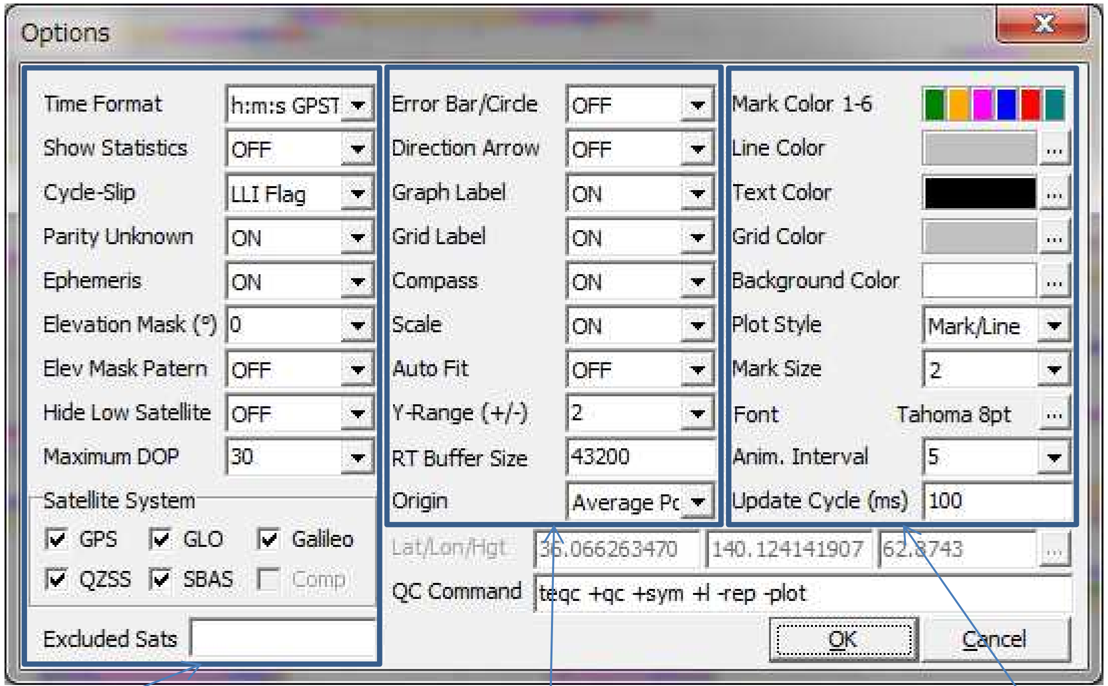
Skyplot





# of Visible Satellites and DOP

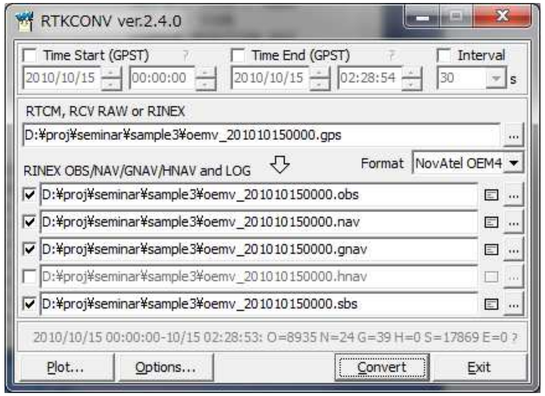
RTKPLOT: Edit - Options



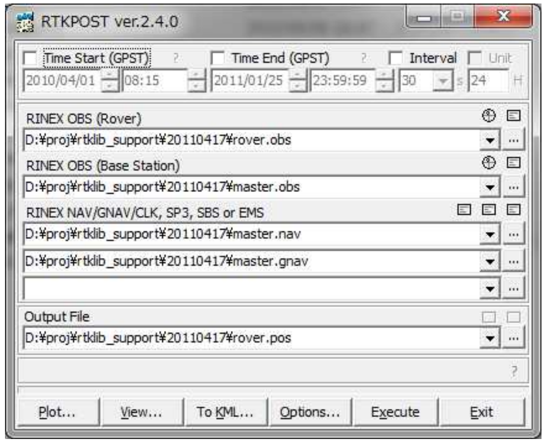
OBS Data Options **Solution Data Options** **Common Options**

RTKCONV - RTKPOST

- **Objective**
  Analysis by standard positioning
- **Program**
  rtklib_2.4.2¥bin¥rtkconv.exe
  rtklib_2.4.2¥bin¥rtkpost.exe
- **Data**
  school¥sample3¥
   oemv_201010150000.gps



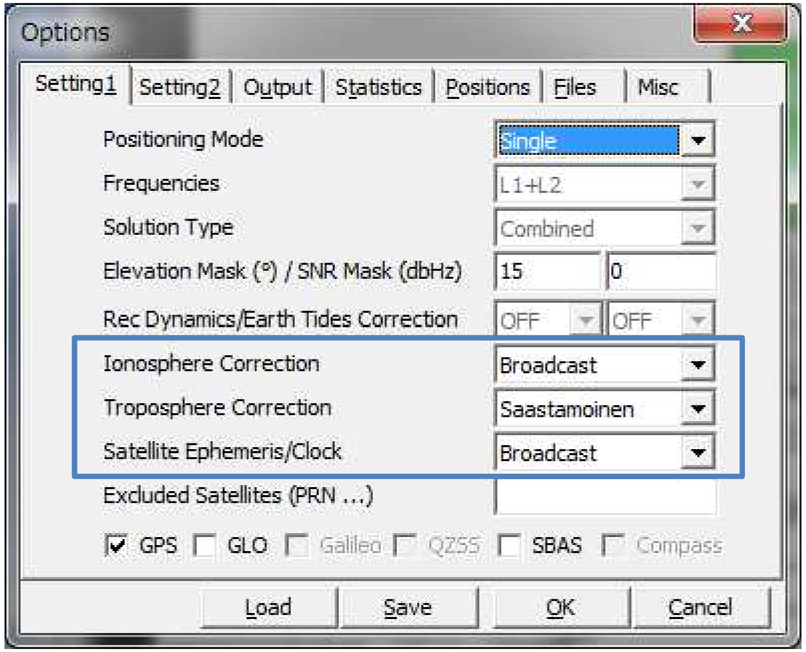
RTKLIB - RTKCONV



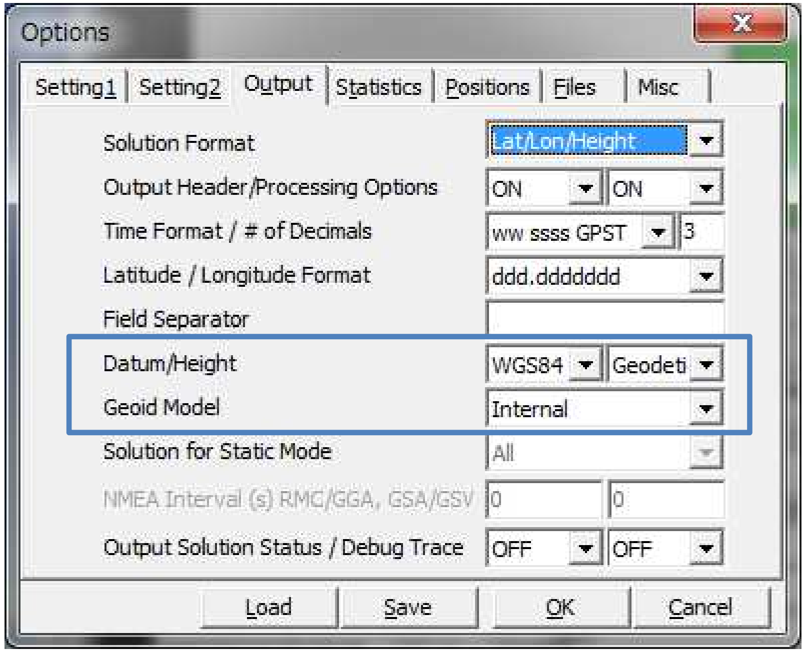
RTKLIB - RTKPOST

RTKPOST - Options

Setting1



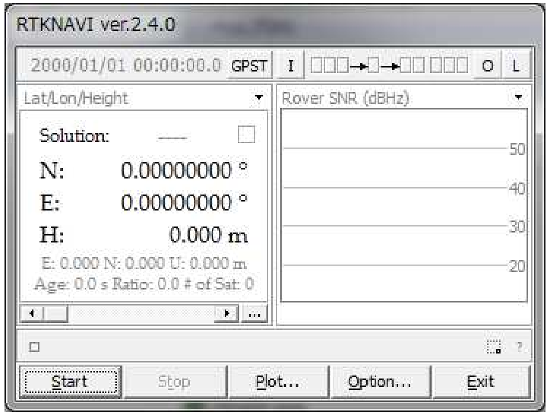
Output



RTKNAVI

- **Objective**
  monitor receiver output data
- **Program**
  rtklib_2.4.2¥bin¥rtknavi.exe
- **Data**
  school¥sample2¥
   ubx_20090515c.ubx (u-blox)
   oemv_2009515c.gps (NovAtel)

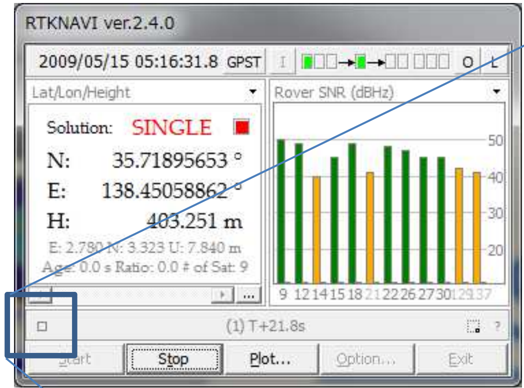
RTKLIB - RTKNAVI



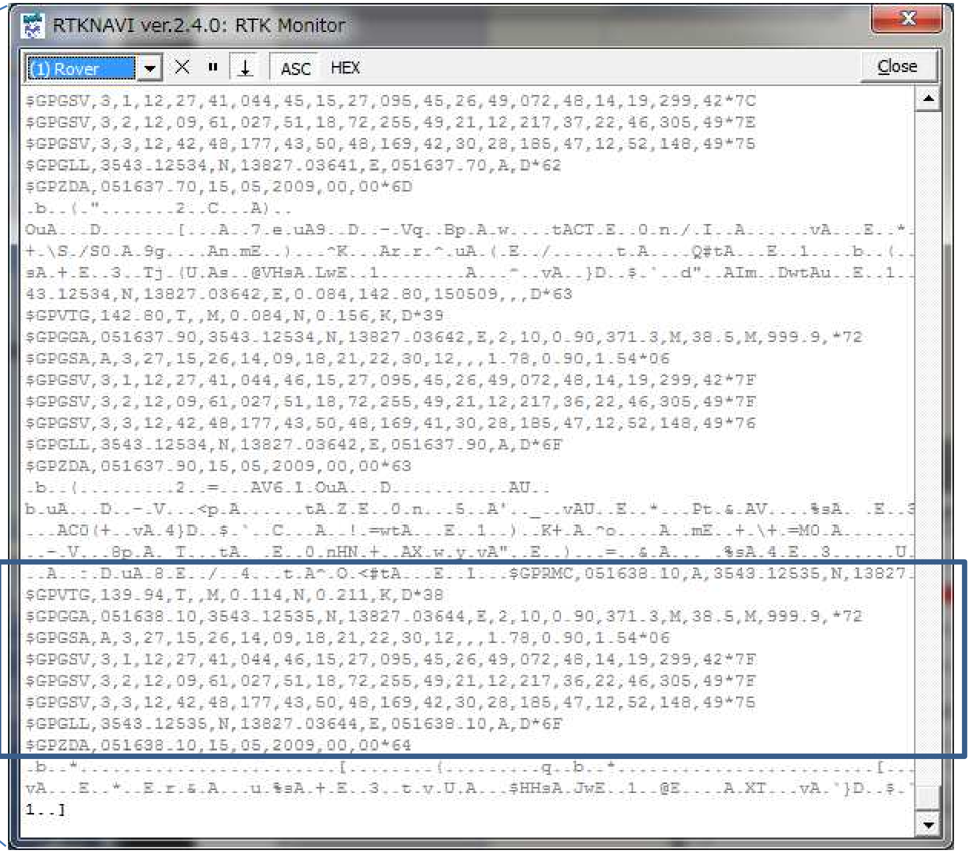
u-blox AEK-4T (LEA-4T)

NovAtel OEMV-3G

Output of u-blox



RTK Monitor



NMEA0183:

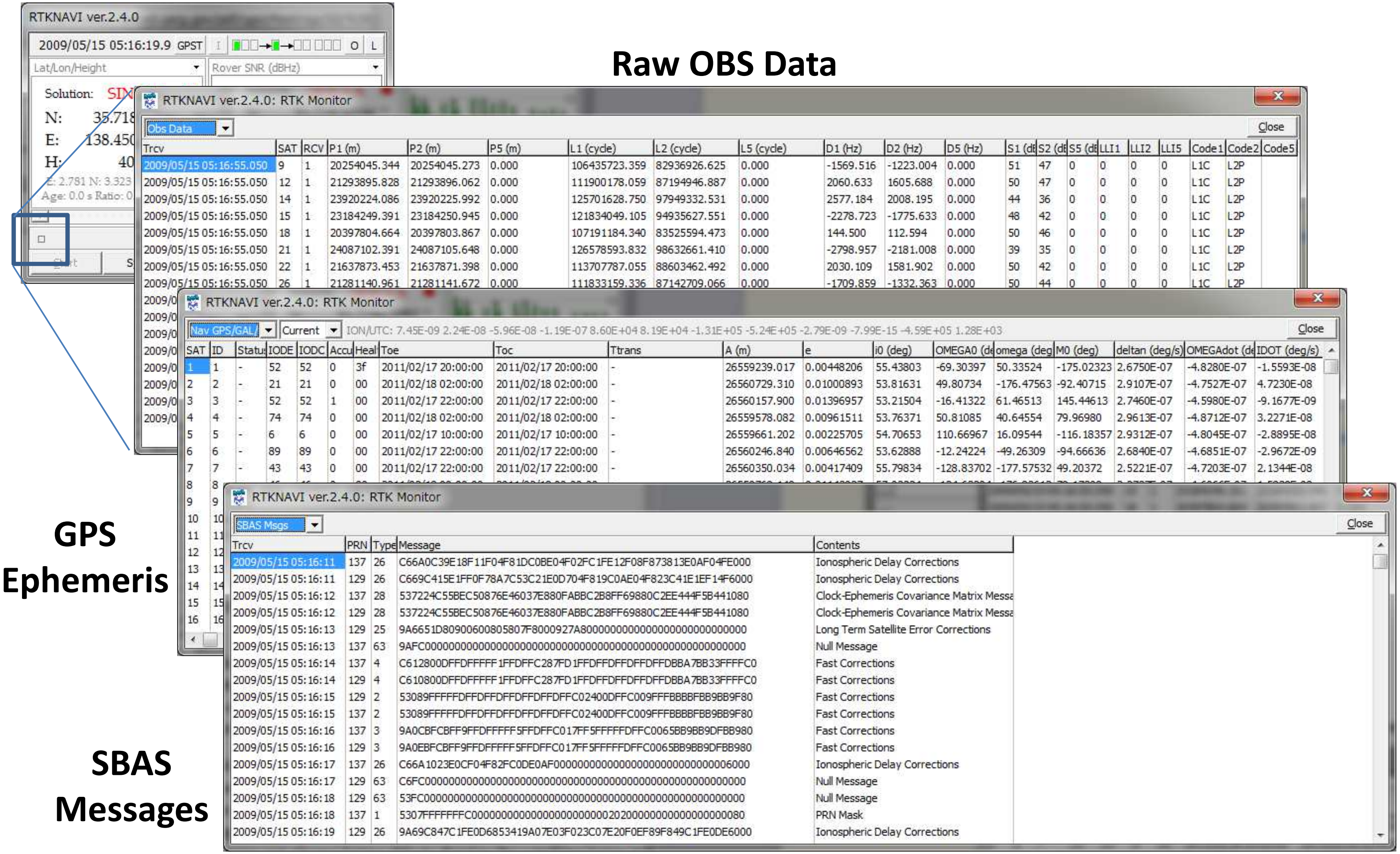
$GPRMC : Recommended minimum data for GPS
$GPGGA : Fix Information
$GPGSA : Overall Satellite Data
$GPGSV : Derailed Satellite Data
$GPGLL : Lat/Lon Data, ...

$GPGGA,123519,4807.038,N,01131.000,E,1,08,0.9,545.4,M,46.9,M,,*47

Time (UTC) — Latitude — Longitude — Quality — # of Sats — HDOP — Altitude — Geoid Height

Output of NovAtel

Raw OBS Data



GPS Ephemeris

SBAS Messages

B-3
Theory of Precise Positioning (1)

GNSS

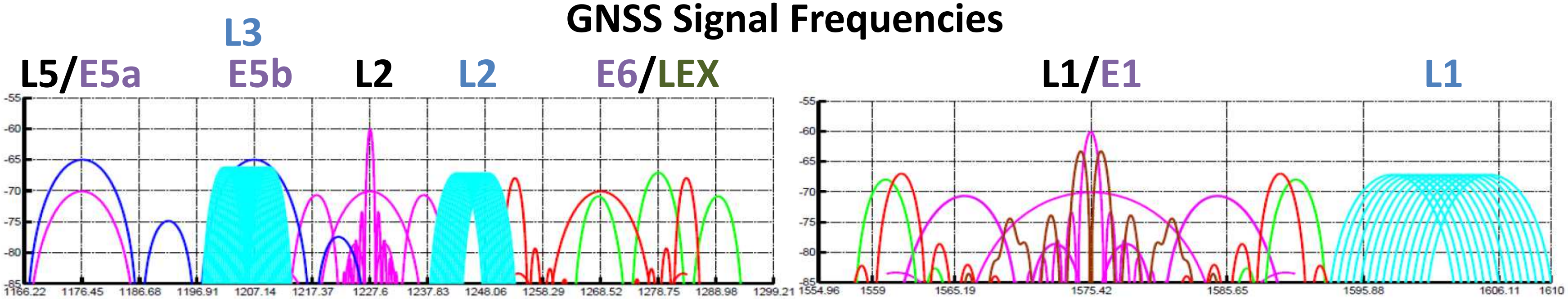
- **GNSS (Global Navigation Satellite System)**
  - **GPS** (US)
  - **GLONASS** (Russia)
  - **Galileo** (EU)
  - **Compass** (China)
- **RNSS (Regional Navigation Satellite System)**
  - **QZSS** (Japan)
  - **IRNSS** (India)
- **SBAS (Satellite Based Augmentation System)**
  - **WAAS, EGNOS, MSAS, SDCM, GAGAN**

GNSS Constellation

Number of Existing/Planned GNSS Satellites

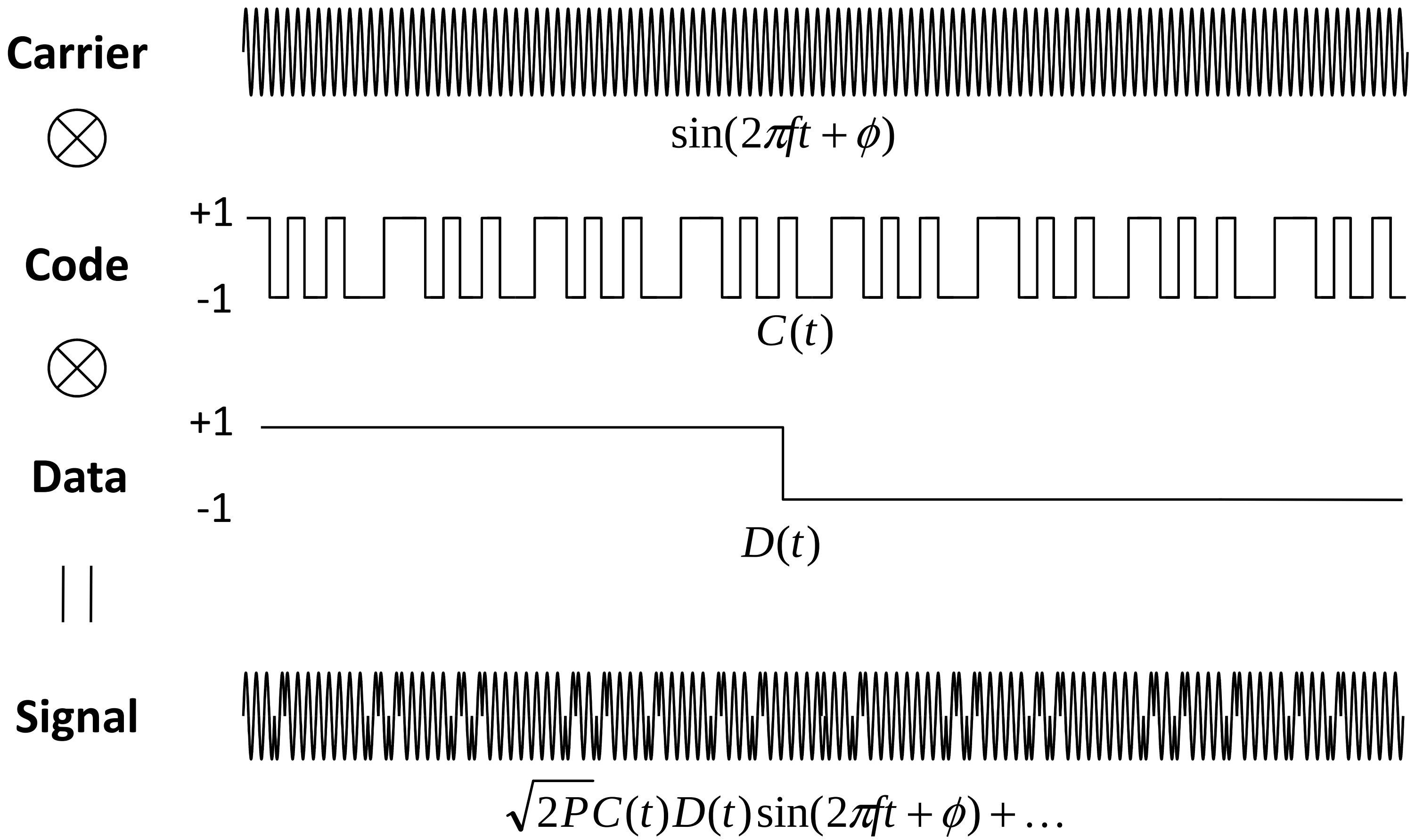
System	2010	2013	2016	2019
GPS	31	31	32	32
GLONASS	23 (+2)	24 (+3)	24 (+3)	24 (+3)
Galileo	0	4	18	27 (+3)
Compass	6	16	30	32 (+3)
QZSS	1	1	4	7
IRNSS	0	1	7	7
SBAS	7	8	11	11
Total	**68**	**91**	**126**	**140**

GNSS Signal Frequencies



(Y.Yang, COMPASS: View on Compatibility and Interoperability, 2009)

GNSS Signal Structure

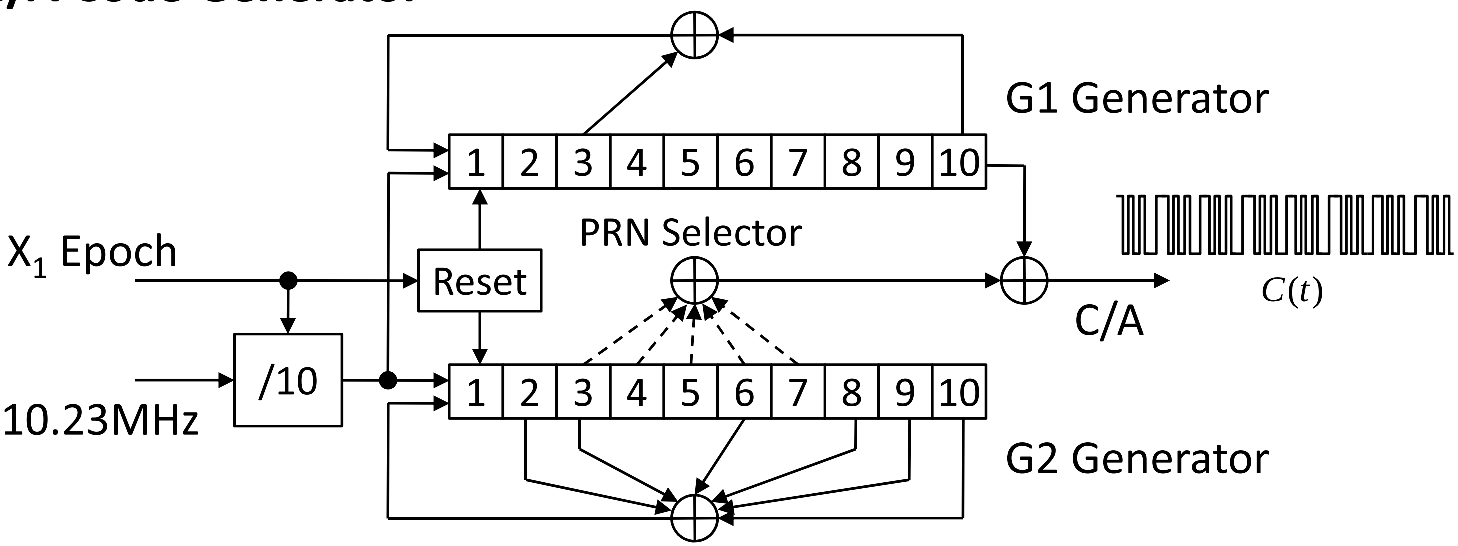


GNSS Signal Specifications

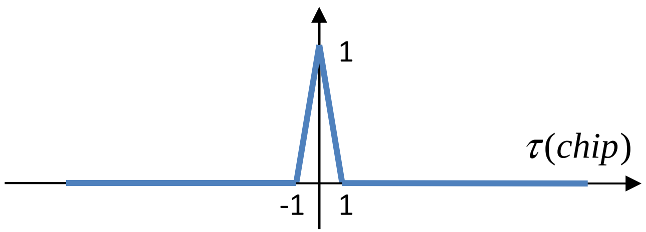
Carrier Freq (MHz)		Code	Modulation	Data Rate	GNSS
L1/E1	1575.42	C/A	BPSK (1)	50 bps	**GPS, QZSS**
				250 bps	**QZSS (L1-SAIF), SBAS**
		P(Y)	BPSK (10)	50 bps	**GPS**
		L1C-d/p	MBOC (6,1,1/11)	-/100 bps	**GPS (III-), Galileo**
		L1C-d/p	BOC (1,1)	-/100 bps	**QZSS**
L1	1602+0.5625K	C/A	BPSK	50 bps	**GLONASS**
L2	1227.60	P(Y)	BPSK (10)	50 bps	**GPS**
		L2C	BPSK (1)	25 bps	**GPS (IIRM-), QZSS**
L2	1246+0.4375K	C/A	BPSK	50 bps	**GLONASS**
L5/E5a	1176.45	L5-I/Q	BPSK (10)	-/100 bps	**GPS (IIF-), QZSS**
		E5a-I/Q	BPSK (10)	-/50 bps	**Galileo**
E5b	1207.14	E5b-I/Q	BPSK (10)	-/250 bps	**Galileo**
E6/LEX	1278.75	E6-I/Q	BPSK (5)	-/1000 bps	**Galileo**
		LEX	BPSK (5)	2000 bps	**QZSS**

Spreading (PRN) Code

GPS C/A Code Generator

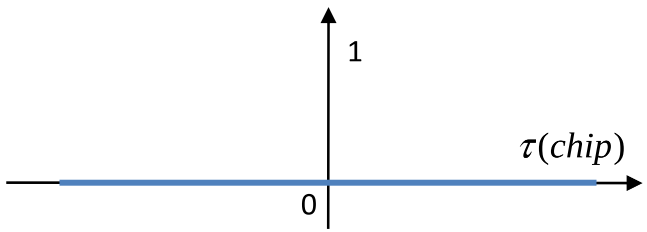


Auto-correlation function



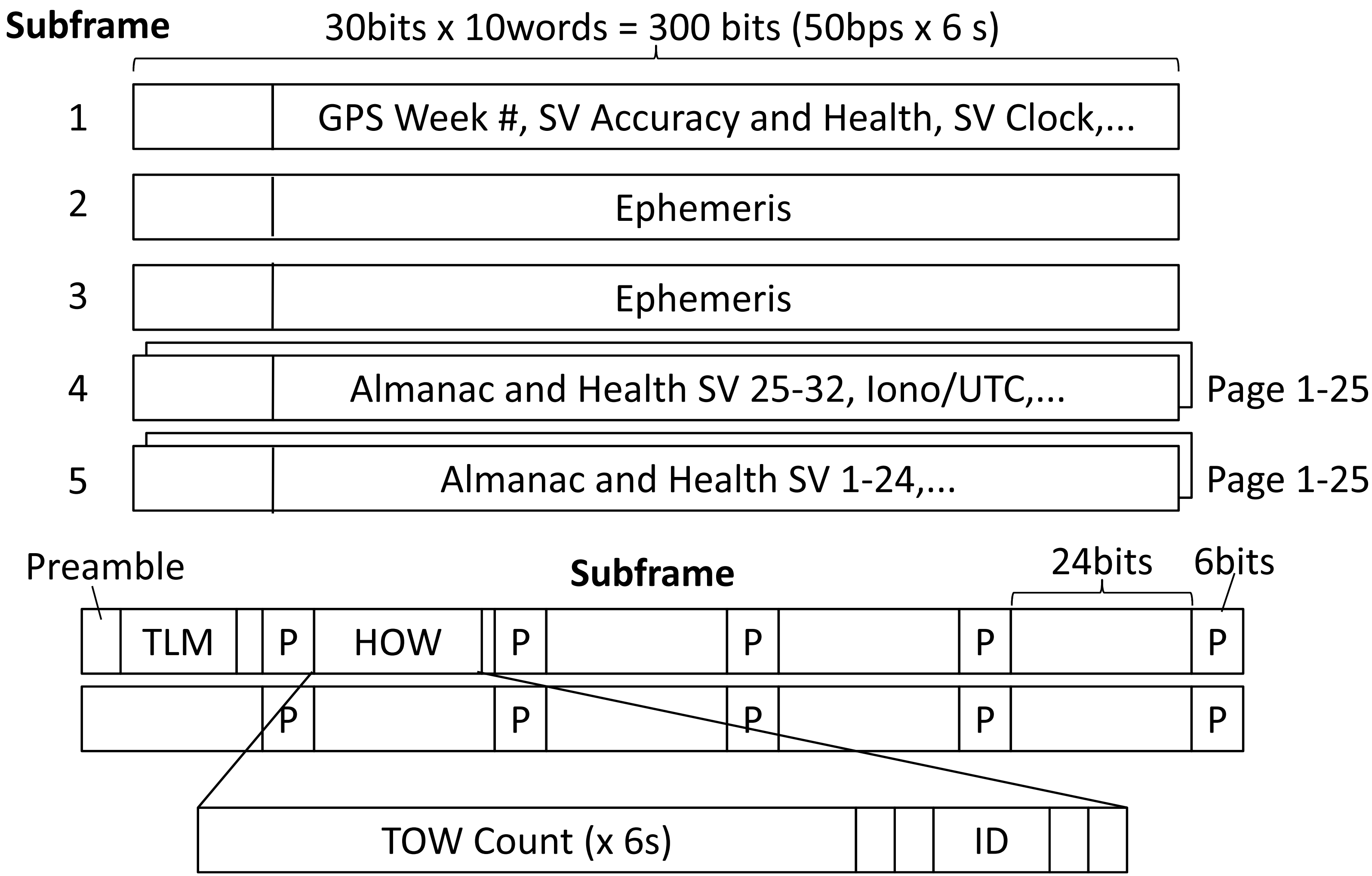
$$R(\tau) = \frac{1}{T}\int_0^T C^i(t)C^i(t-\tau)dt$$

Cross-correlation function



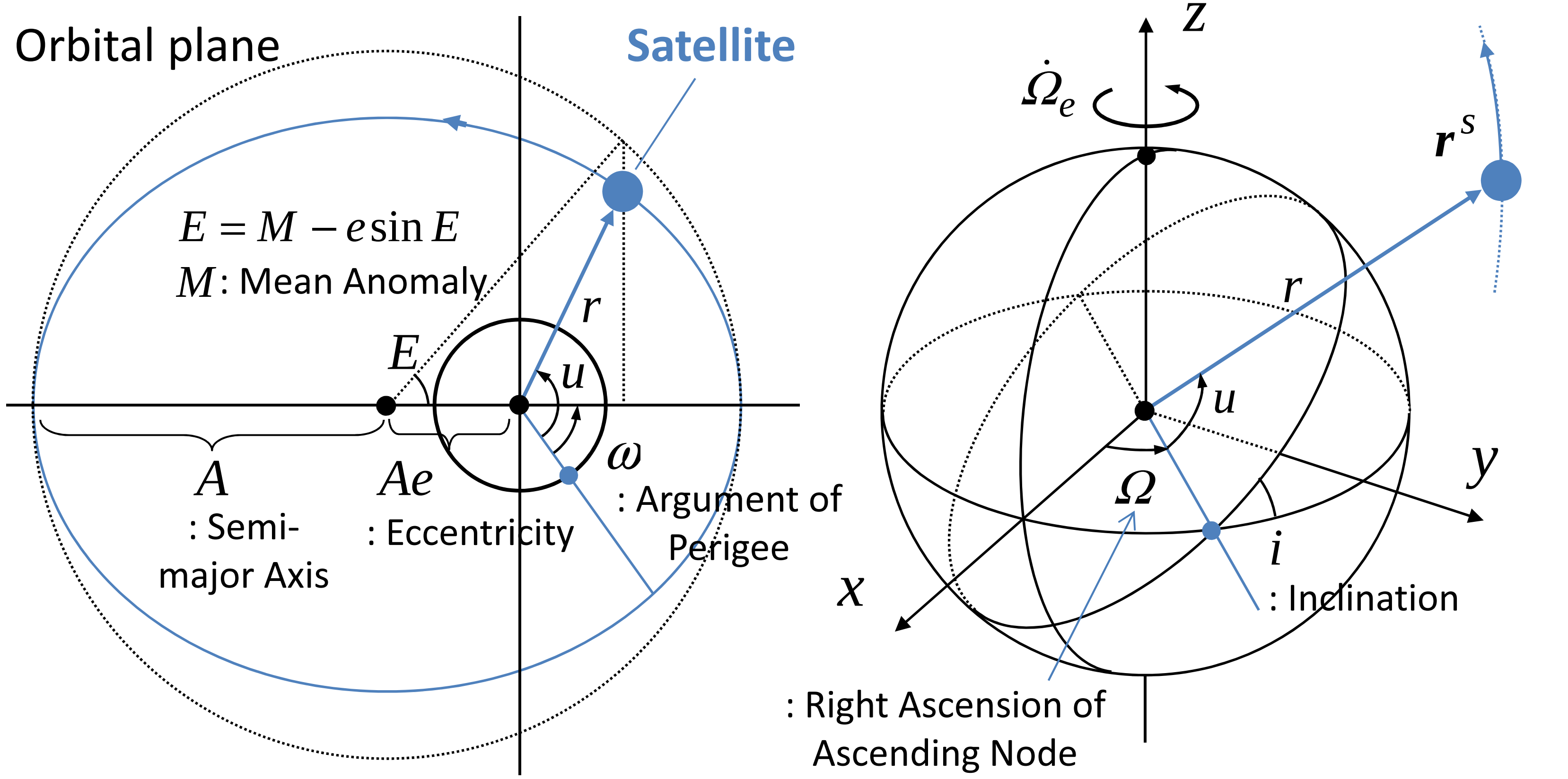
$$R(\tau) = \frac{1}{T}\int_0^T C^i(t)C^j(t-\tau)dt \quad (i \neq j)$$

Navigation Data



Ephemeris

$$M_0, \Delta n, e, \sqrt{A}, \Omega_0, i_0, \omega, \dot{\Omega}, IDOT, C_{uc}, C_{us}, C_{rc}, C_{rs}, C_{ic}, C_{is}, T_{oe}$$



Satellite Position by Ephemeris

$$t_k = t - t_{oe}$$

$$n = \sqrt{\mu / A^3} + \Delta n$$

$$M = M_0 + n t_k$$

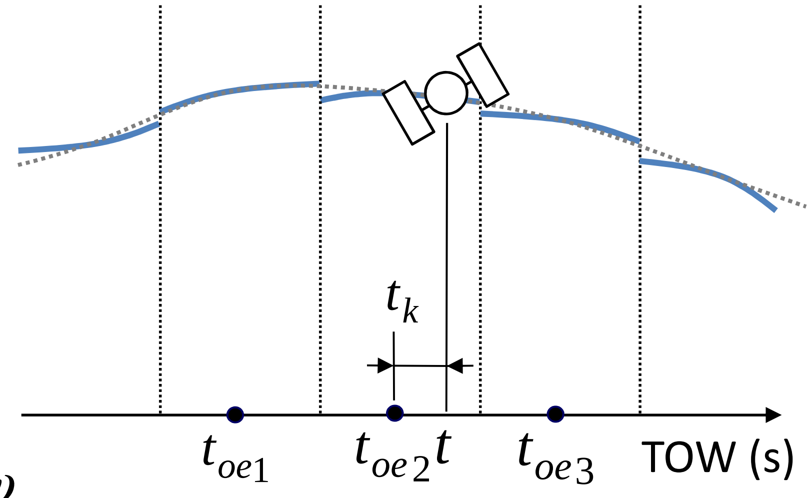
$$E = M + e \sin E$$: Kepler Equation

$$\phi = ATAN2(\sqrt{1-e^2}\sin E, \cos E - e) + \omega$$

$$\begin{pmatrix} u \\ r \\ i \end{pmatrix} = \begin{pmatrix} \phi \\ A(1 - e\cos E) \\ i_0 + IDOT\ t_k \end{pmatrix} + \begin{pmatrix} C_{us} & C_{uc} \\ C_{rs} & C_{rc} \\ C_{is} & C_{ic} \end{pmatrix} \begin{pmatrix} \sin 2\phi \\ \cos 2\phi \end{pmatrix}$$

$$\Omega = \Omega_0 + (\dot{\Omega} - \omega_e) t_k - \omega_e t_{oe}$$

$$\boldsymbol{r}^s(t) = \boldsymbol{R}_z(-\Omega)\boldsymbol{R}_x(-i)(r\cos u, r\sin u, 0)^T$$



$$\boldsymbol{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

$$\boldsymbol{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\boldsymbol{R}_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

SV Clock Parameters

$$a_{f0}, a_{f1}, a_{f2}, T_{GD}, t_{oc}$$

Satellite Clock Bias

$$dT(t) = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \Delta t_{rel} + \Delta t_{GD}$$

Relativity Correction:

$$\Delta t_{rel} = \frac{-2\sqrt{\mu A} e \sin E}{c^2}$$

Group Delay Correction:

$$\Delta t_{GD} = \begin{cases} -T_{GD} & (L1) \\ -\gamma T_{GD} & (L2) \quad (\gamma = f_1^2 / f_2^2) \\ 0 & (LC) \end{cases}$$

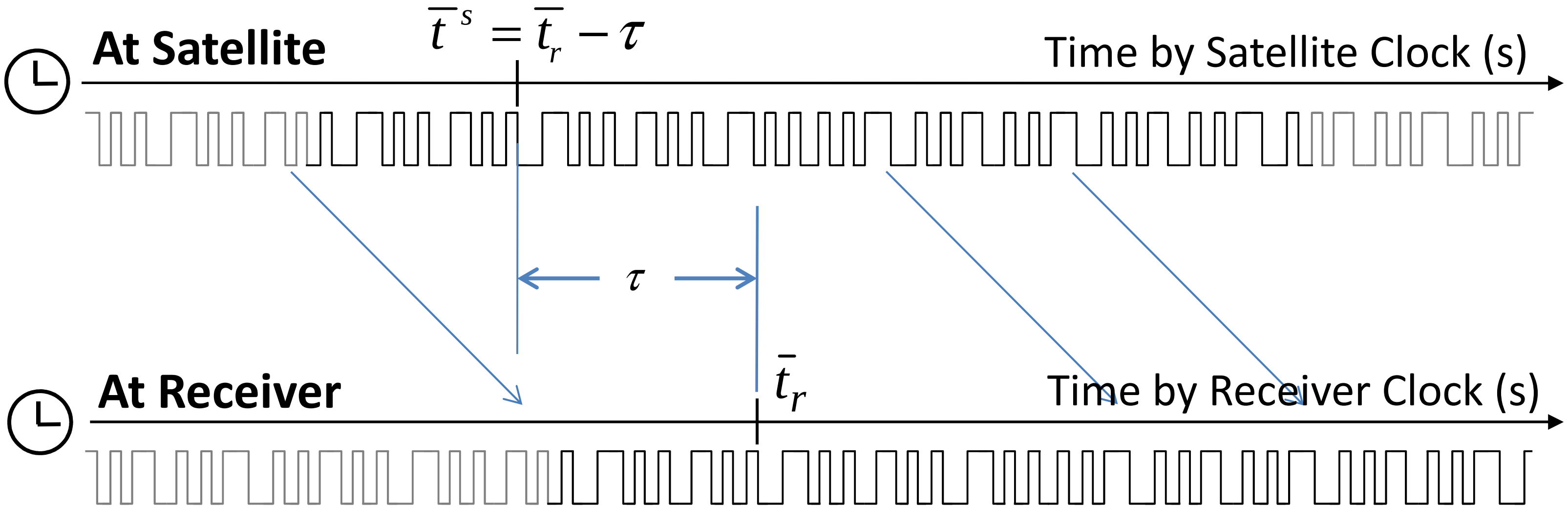
Pseudorange

Definition:

$$P_r^s \equiv c\tau = c(\overline{t}_r - \overline{t}^{\,s})$$

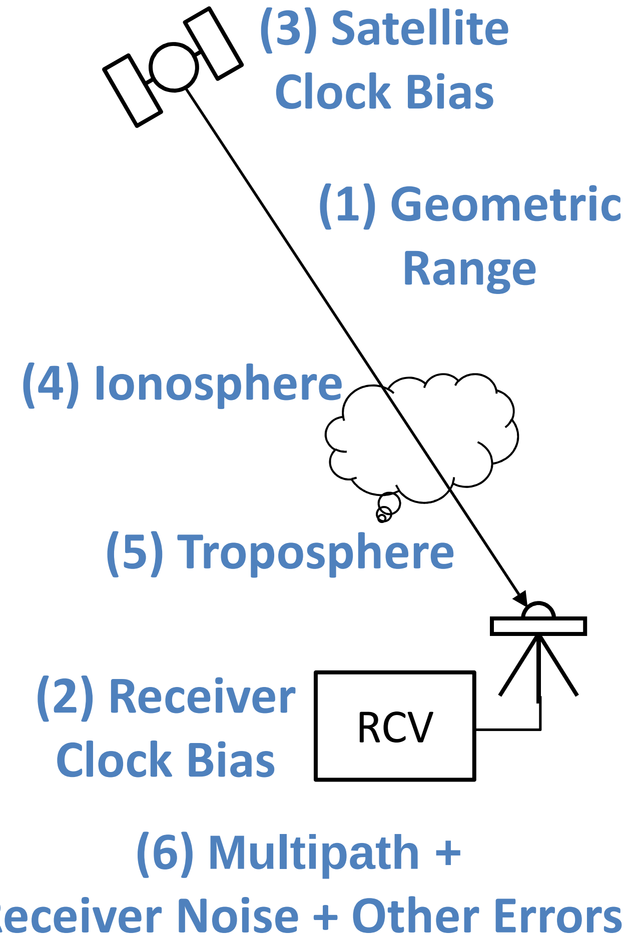
(m)

The pseudo-range (PR) is the distance from the receiver antenna to the satellite antenna including receiver and satellite clock offsets (and other biases, such as atmospheric delays) *(RINEX 2.10)*

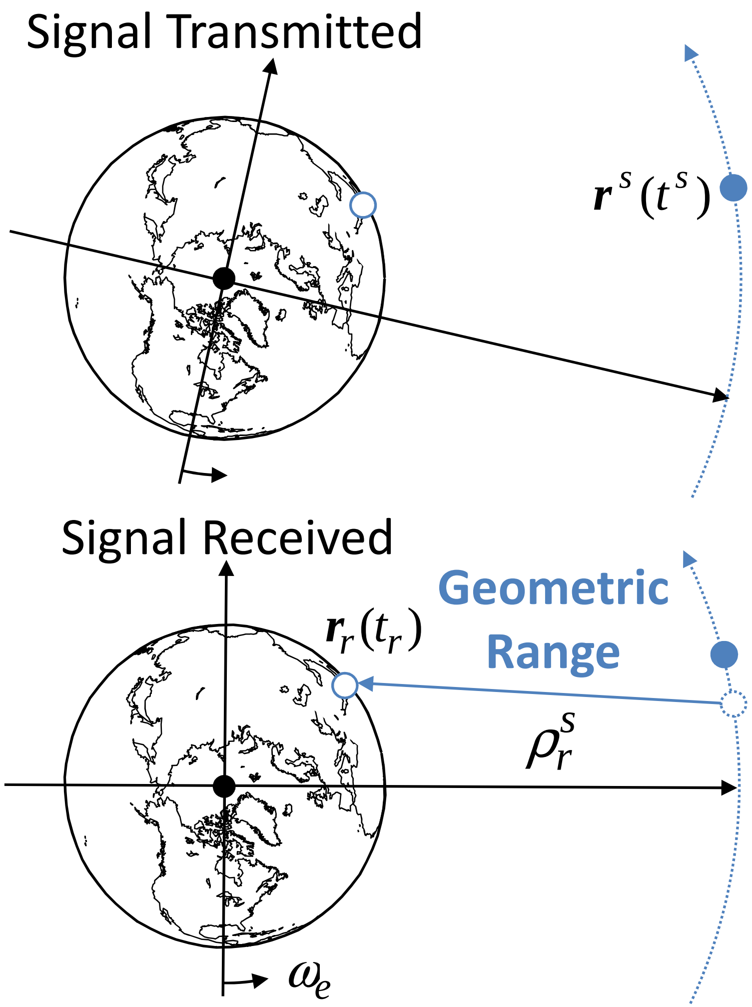


Pseudorange Model

$$
\begin{aligned}
P_r^s &\equiv c\tau \\
&= c(\overline{t}_r - \overline{t}^s) \\
&= c((t_r + dt) - (t^s + dT^s)) + \varepsilon_P \\
&= c(t_r - t^s) + c(dt_r - dT^s) + \varepsilon_P \\
&= (\rho_r^s + I_r^s + T_r^s) + c(dt_r - dT^s) + \varepsilon_P \\
&= \underbrace{\rho_r^s}_{(1)} + c(\underbrace{dt_r}_{(2)} - \underbrace{dT^s}_{(3)}) + \underbrace{I_r^s}_{(4)} + \underbrace{T_r^s}_{(5)} + \underbrace{\varepsilon_P}_{(6)}
\end{aligned}
$$



Geometric Range



Signal Transmission Time

$$t^s = \bar{t}_r - P_r^s / c - dT(t^s)$$

(1)

$$\rho_r^s = \left\| \boldsymbol{U}(t_r)\boldsymbol{r}_r(t_r) - \boldsymbol{U}(t^s)\boldsymbol{r}^s(t^s) \right\|$$

(2)

$$\rho_r^s \approx \left\| \boldsymbol{r}_r(t_r) - \boldsymbol{R}_z(\omega_e(t_r - t^s))\boldsymbol{r}^s(t^s) \right\|$$

(3)

$$\rho_r^s \approx \left\| \boldsymbol{r}_r(t_r) - \boldsymbol{R}_z(\omega_e \rho_r^s / c)\boldsymbol{r}^s(t^s) \right\|$$

(4)

$$\rho_r^s \approx \left\| \boldsymbol{r}_r(t_r) - \boldsymbol{r}^s(t^s) \right\| + \frac{\omega_e(x^s y_r - y^s x_r)}{c}$$

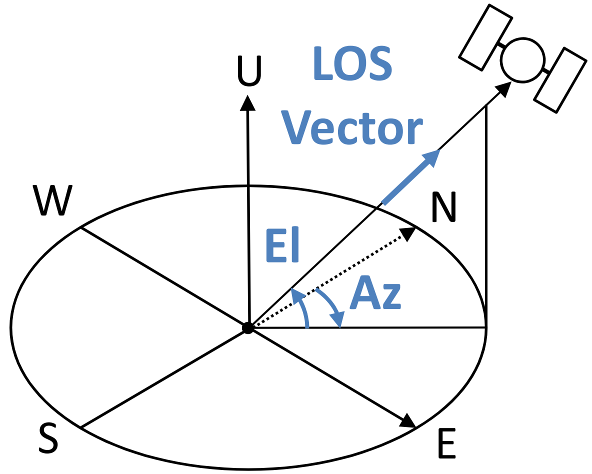
Sagnac Effect Correction

LOS (Line-of-Sight) Vector

LOS Vector:

$$\boldsymbol{e}_r^s = \frac{\boldsymbol{r}^s - \boldsymbol{r}_r}{\left\| \boldsymbol{r}^s - \boldsymbol{r}_r \right\|}, \quad \boldsymbol{e}_{r,enu}^s = \boldsymbol{E}_{ecef \to enu} \boldsymbol{e}_r^s = (e_e, e_n, e_u)^T$$

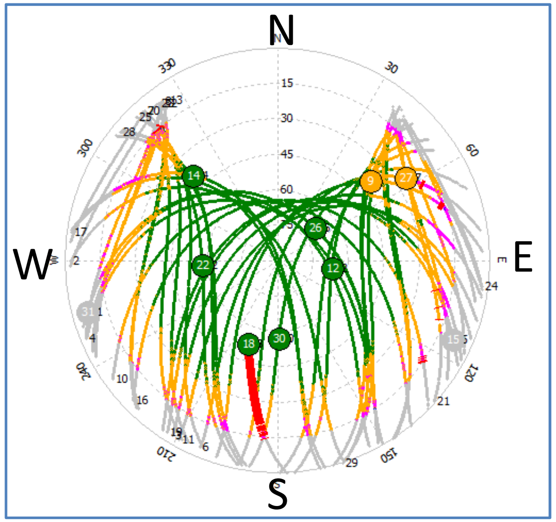
$$\boldsymbol{E}_{ecef \to enu} = \begin{pmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{pmatrix}$$



Satellite Azimuth/Elevation Angle:

$$Az = \mathrm{ATAN2}(e_e, e_n)$$

$$El = \arcsin e_u$$



Ionospheric Model

$$\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2, \beta_3$$

Klobuchar Model:

$$\psi = 0.0137/(El + 0.11) - 0.022$$

$$\phi_i = \phi + \psi \cos Az$$

$$\lambda_i = \lambda + \psi \sin Az / \cos \phi_i$$

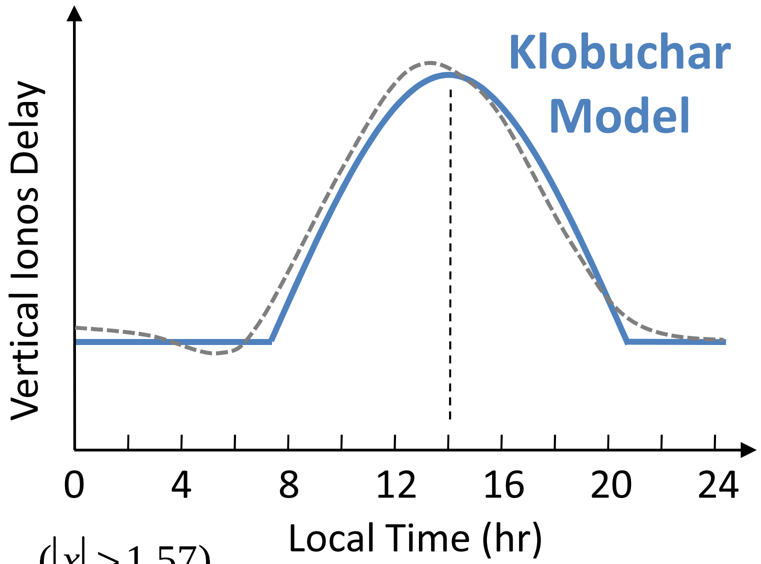
$$\phi_m = \phi_i + 0.064 \cos(\lambda_i - 1.617)$$

$$t = 4.32 \times 10^4 \lambda_i + t$$

$$F = 1.0 + 16.0 \times (0.53 - El)^3$$

$$x = 2\pi(t - 50400) / \sum_{n=0}^{3} \beta_n {\phi_m}^n$$

$$I = \begin{cases} F \times 5 \times 10^{-9} & (|x| > 1.57) \\ F \times \left(5 \times 10^{-9} + \sum_{n=1}^{4} \alpha_n {\phi_m}^n \times \left(1 - \frac{x^2}{2} + \frac{x^4}{24} \right) \right) & (|x| \le 1.57) \end{cases}$$



Troposphere Model

Standard Atmosphere:

$$p = 1013.25 \times (1 - 2.2557 \times 10^{-5} H)^{5.2568}$$

$$T = 15.0 - 6.5 \times 10^{-3} H + 273.15$$

$$e = 6.108 \times \exp\left\{\frac{17.15\mathrm{T} - 4684.0}{\mathrm{T} - 38.45}\right\} \times \frac{h_{rel}}{100}$$

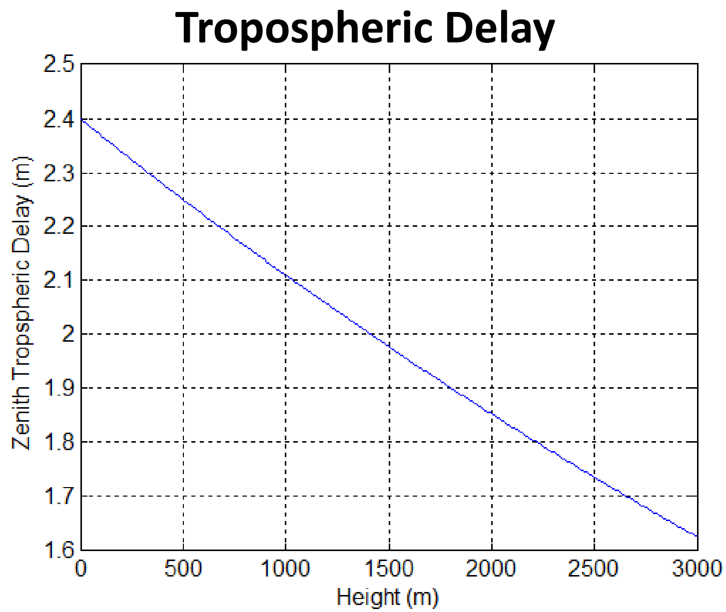
p : Pressure (hPa)

H : Geopotential Height (m)

T : Temperature (K)

e : Partial Pressure of WV (hPa)

h_{rel} : Relative Humidity (%)



h_{rel}=50%, z=90°

Saastamoinen Model:

$$T_r^s = \frac{0.002277}{\cos z}\left\{p + \left(\frac{1255}{T} + 0.05\right)e - \tan^2 z\right\}$$

(z : Zenith Angle)

LSE (Least Square Estimation)

Measurement Equation:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$$

$\boldsymbol{y}$: Measurement vector $\boldsymbol{H}$: Design matrix

$\boldsymbol{x}$: Parameter vector $\boldsymbol{v}$: Residual vector

$$J_{LS} = v_1^2 + v_2^2 + \ldots + v_m^2 = \boldsymbol{v}^T\boldsymbol{v} = (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})^T(\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})$$

$$= \boldsymbol{y}^T\boldsymbol{y} - \boldsymbol{y}^T\boldsymbol{H}\boldsymbol{x} - \boldsymbol{x}^T\boldsymbol{H}^T\boldsymbol{y} + \boldsymbol{x}^T\boldsymbol{H}^T\boldsymbol{H}\boldsymbol{x} \rightarrow \min$$

$$\frac{\partial J_{LS}}{\partial \boldsymbol{x}} = \boldsymbol{0}^T - \boldsymbol{y}^T\boldsymbol{H} - (\boldsymbol{H}^T\boldsymbol{y})^T + (\boldsymbol{H}^T\boldsymbol{H}\boldsymbol{x})^T + \boldsymbol{x}^T\boldsymbol{H}^T\boldsymbol{H}$$

$$= -2\boldsymbol{y}^T\boldsymbol{H} + 2\boldsymbol{x}^T\boldsymbol{H}^T\boldsymbol{H} = \boldsymbol{0}$$

Normal Equation (NEQ):

$$\boldsymbol{H}^T\boldsymbol{H}\hat{\boldsymbol{x}} = \boldsymbol{H}^T\boldsymbol{y} \rightarrow \hat{\boldsymbol{x}} = (\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{y}$$

Weighted LSE:

$$\hat{\boldsymbol{x}} = (\boldsymbol{H}^T\boldsymbol{W}\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{W}\boldsymbol{y} \quad (J_{WLS} = \boldsymbol{v}^T\boldsymbol{W}\boldsymbol{v} \rightarrow \min)$$

Non-linear LSE

Measurement Equation:

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{v}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{h}(\boldsymbol{x}_0) + \boldsymbol{H}(\boldsymbol{x} - \boldsymbol{x}_0) + \ldots$$: Taylor Polynomial

$$\boldsymbol{y} \approx \boldsymbol{h}(\boldsymbol{x}_0) + \boldsymbol{H}(\boldsymbol{x} - \boldsymbol{x}_0) + \boldsymbol{v}$$

$$\boldsymbol{y} - \boldsymbol{h}(\boldsymbol{x}_0) = \boldsymbol{H}(\boldsymbol{x} - \boldsymbol{x}_0) + \boldsymbol{v}$$

$$\boldsymbol{H}^T \boldsymbol{H}(\hat{\boldsymbol{x}} - \boldsymbol{x}_0) = \boldsymbol{H}^T(\boldsymbol{y} - \boldsymbol{h}(\boldsymbol{x}_0))$$

$$\left(\boldsymbol{H} = \left.\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{x}_0} \right)$$

Matrix Partials

$$\hat{\boldsymbol{x}} = \boldsymbol{x}_0 + (\boldsymbol{H}^T \boldsymbol{H})^{-1} \boldsymbol{H}^T(\boldsymbol{y} - \boldsymbol{h}(\boldsymbol{x}_0))$$

Iterative Solution (Gauss-Newton):

$$\hat{\boldsymbol{x}}_{i+1} = \hat{\boldsymbol{x}}_i + (\boldsymbol{H}^T \boldsymbol{H})^{-1} \boldsymbol{H}^T(\boldsymbol{y} - \boldsymbol{h}(\hat{\boldsymbol{x}}_i))$$

$$\hat{\boldsymbol{x}} = \lim_{i \to \infty} \hat{\boldsymbol{x}}_i$$

Navigation Processing

$$\boldsymbol{x} = (\boldsymbol{r}_r{}^T, cdt)^T, \quad \boldsymbol{y} = (P_r^{s_1}, P_r^{s_2}, P_r^{s_3}, \ldots, P_r^{s_m})^T$$

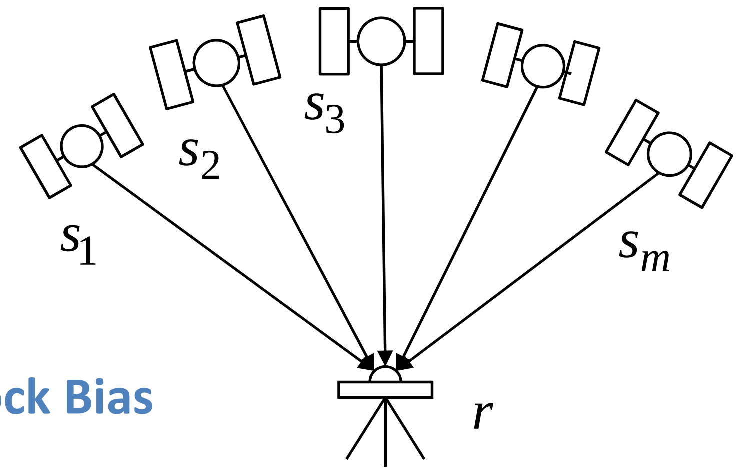
$$\boldsymbol{h}(\hat{\boldsymbol{x}}) = \begin{pmatrix} \rho_r^{s_1} + c\hat{d}t - cdT^{s_1} + I_r^{s_1} + T_r^{s_1} \\ \rho_r^{s_2} + c\hat{d}t - cdT^{s_2} + I_r^{s_2} + T_r^{s_2} \\ \rho_r^{s_3} + c\hat{d}t - cdT^{s_3} + I_r^{s_3} + T_r^{s_3} \\ \vdots \\ \rho_r^{s_m} + c\hat{d}t - cdT^{s_m} + I_r^{s_m} + T_r^{s_m} \end{pmatrix} \quad \boldsymbol{H} = \begin{pmatrix} -\boldsymbol{e}_r^{s_1}{}^T & 1 \\ -\boldsymbol{e}_r^{s_2}{}^T & 1 \\ -\boldsymbol{e}_r^{s_3}{}^T & 1 \\ \vdots & \vdots \\ -\boldsymbol{e}_r^{s_m}{}^T & 1 \end{pmatrix}$$

$$\hat{\boldsymbol{x}}_0 = (0,0,0,0)^T$$

$$\hat{\boldsymbol{x}}_{i+1} = \hat{\boldsymbol{x}}_i + (\boldsymbol{H}^T \boldsymbol{H})^{-1} \boldsymbol{H}^T (\boldsymbol{y} - \boldsymbol{h}(\hat{\boldsymbol{x}}_i))$$

$$\hat{\boldsymbol{x}} = \lim_{i \to \infty} \hat{\boldsymbol{x}}_i = \underline{(\hat{\boldsymbol{r}}_r{}^T, c\hat{d}t)^T}$$

Single-Point Solution + Receiver Clock Bias



Solution Convergence

Estimated Parameters in LSE Iteration Loop

i	x (m)	y (m)	z (m)	cdt (m)
(0) X=	0.0000000	0.0000000	0.0000000	0.0000000
(1) X=	-4739338.8790644	3968053.3426383	4470195.0681293	1290751.6350707
(2) X=	-3990084.5939062	3334559.7805777	3763444.6383541	50195.3310677
(3) X=	-3957255.7455862	3310242.1098583	3737755.6233736	510.7878812
(4) X=	-3957205.2229884	3310203.7001970	3737718.0508664	432.5789153
(5) X=	-3957205.1820501	3310203.6651692	3737718.0078941	432.4910365
(6) X=	-3957205.1820116	3310203.6651363	3737718.0078537	432.4909539
(7) X=	-3957205.1820116	3310203.6651363	3737718.0078536	432.4909538
(8) X=	-3957205.1820116	3310203.6651363	3737718.0078536	432.4909538
(9) X=	-3957205.1820116	3310203.6651363	3737718.0078536	432.4909538
(10)X=	-3957205.1820116	3310203.6651363	3737718.0078536	432.4909538

2001/1/1 0:00:00, TKSB, processed by RTKLIB 2.2.1, n=8

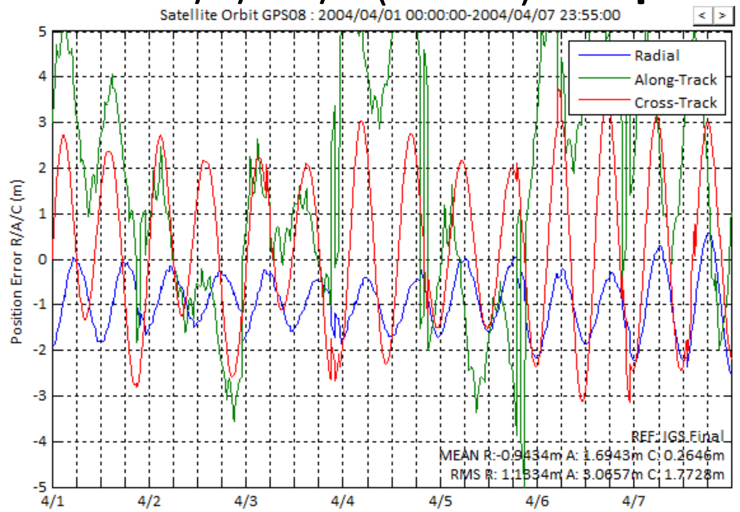
Error Sources and DOP

- **Error sources of Standard positioning**
  - Ephemeris/SV Clock Error
  - Ionospheric Model Error
  - Tropospheric Model Error
  - Multipath
  - Receiver Noise
  - Other Errors
  - S/A (Selective Availability)
- **Satellites-Receiver Geometry**
  - DOP (Dilution of Precision)

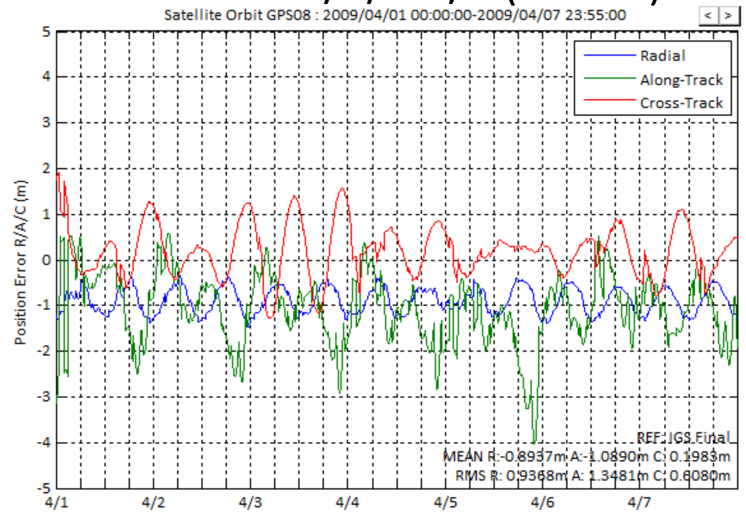
Ephemeris/SV Clock Error

Ephemeris Error

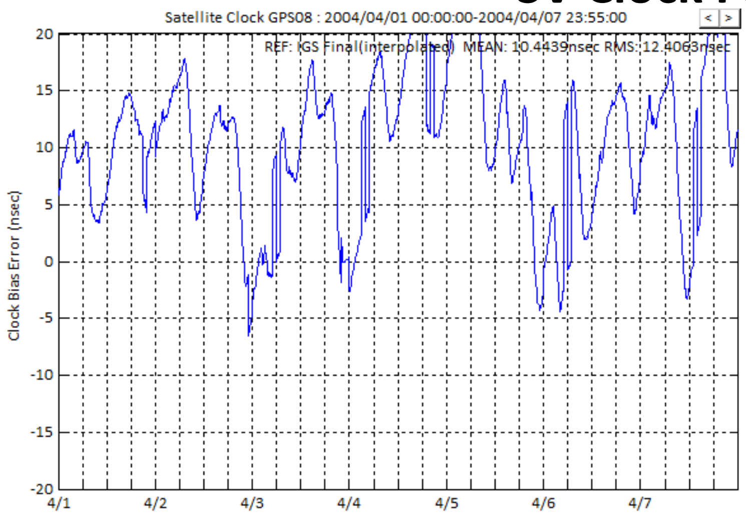
2004/4/1-4/7 (PRN08)

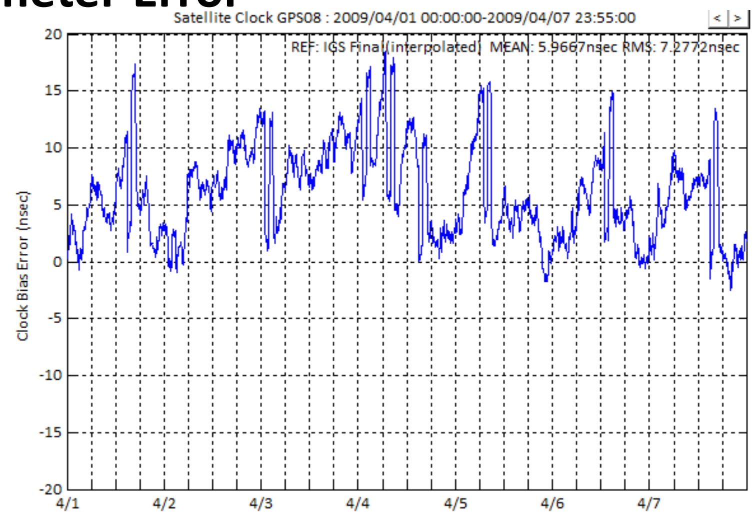


2009/4/1-4/7 (PRN08)

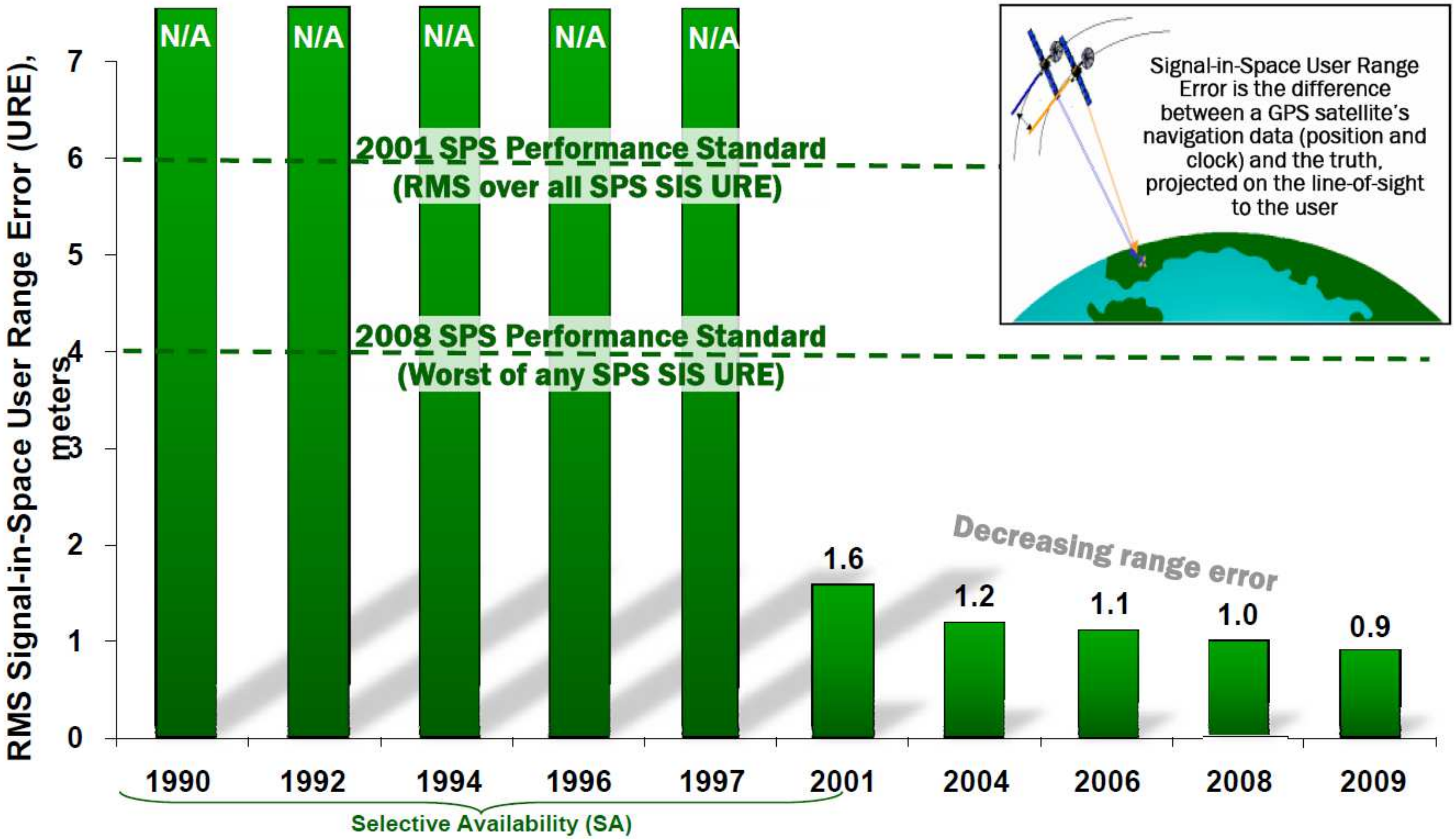


SV Clock Parameter Error



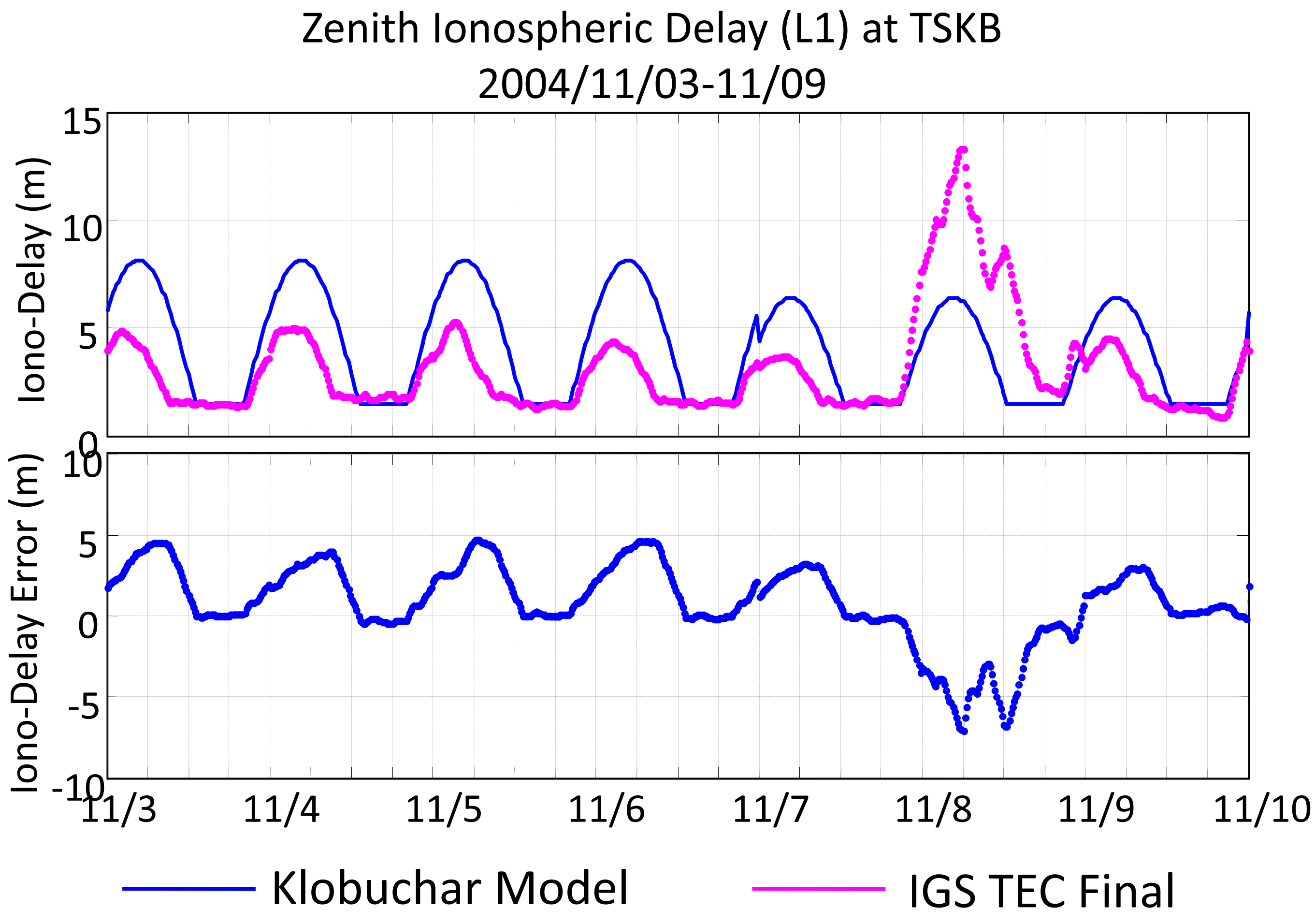


GPS SIS-URE



L.S.Steiner, GPS Program Update to CGSIC 2010, Sep 21, 2010

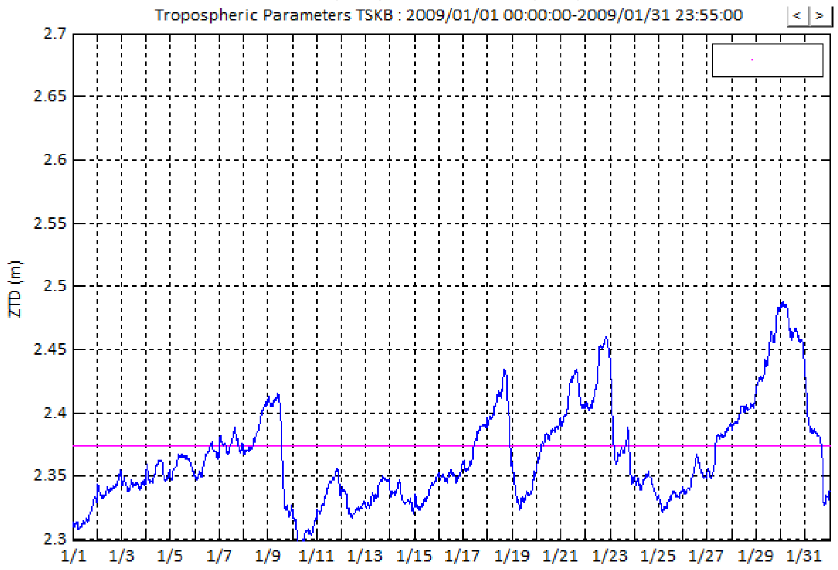
Ionospheric Model Error



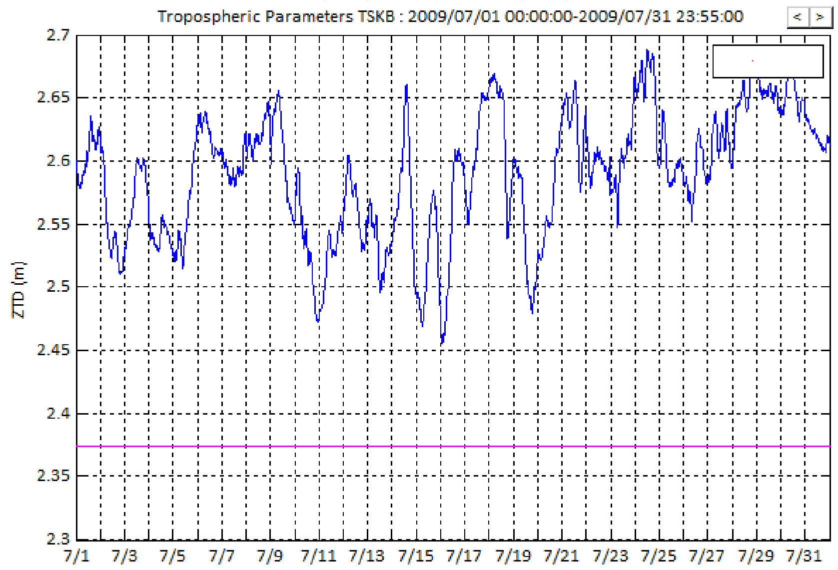
Tropospheric Model Error

ZTD (Zenith Total Delay) at TSKB

2009/1/1-2009/1/31



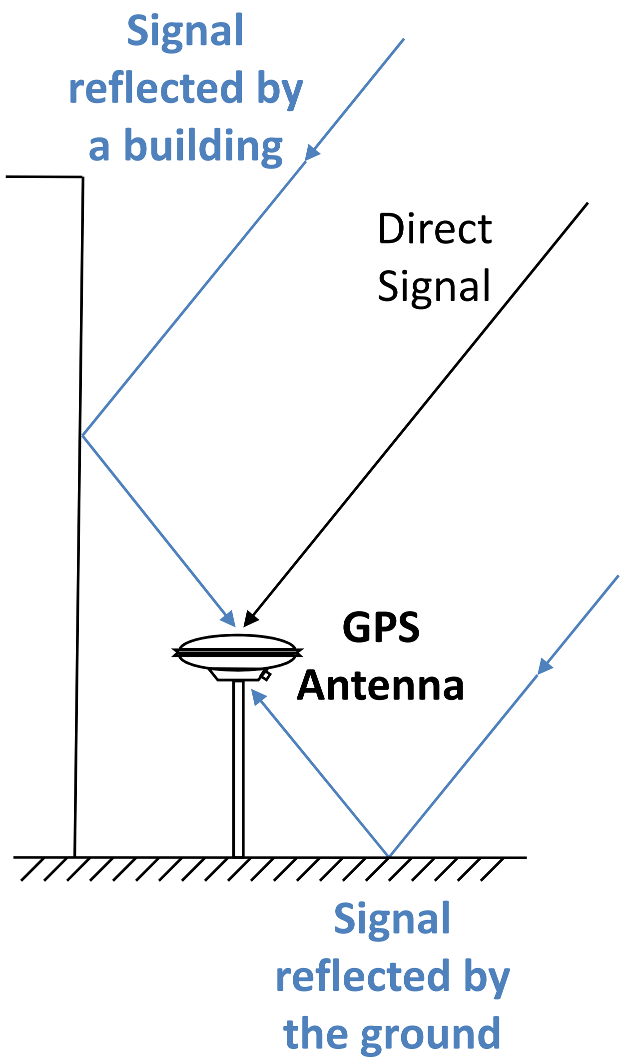
2009/7/1-2009/7/31



Saastamoinen Model

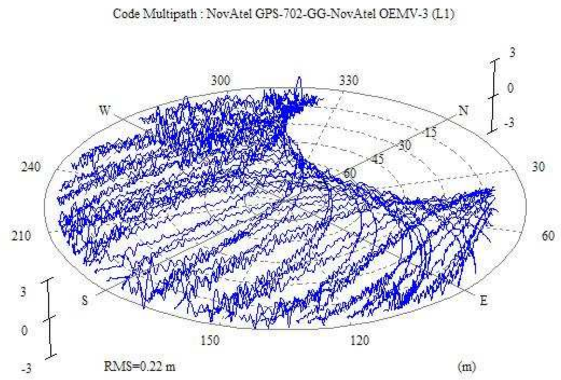
Estimated by PPP

Multipath

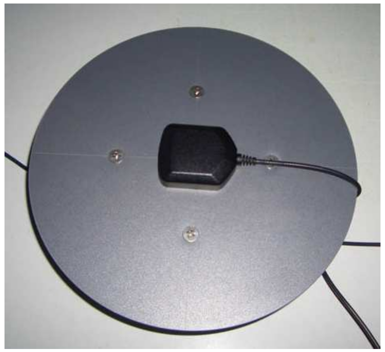


Geodetic-Grade Antenna

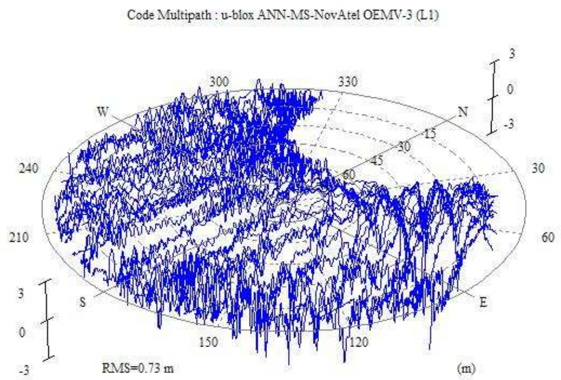
NovAtel
GPS-702-GG



Consumer-Grade Antenna



u-blox ANN-MS



DOP

GDOP, PDOP, HDOP, VDOP

$$GDOP = \sqrt{q_{ee} + q_{nn} + q_{uu} + q_{tt}}$$

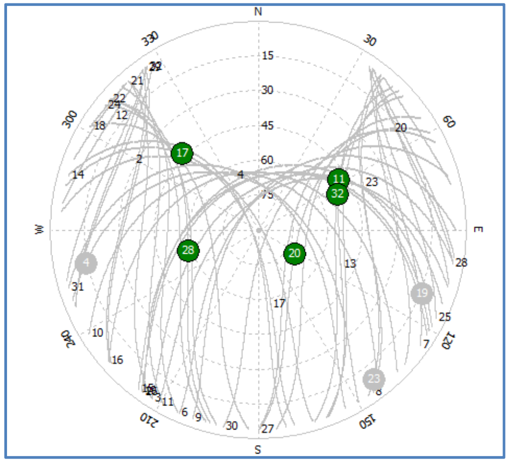
$$PDOP = \sqrt{q_{ee} + q_{nn} + q_{uu}}$$

$$HDOP = \sqrt{q_{ee} + q_{nn}}$$

$$VDOP = \sqrt{q_{uu}}$$

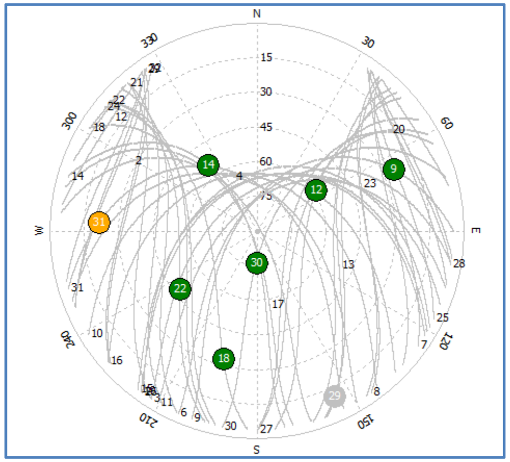
$$\boldsymbol{Q} = (\boldsymbol{H}^T \boldsymbol{H})^{-1} = \begin{pmatrix} q_{ee} & q_{en} & q_{eu} & q_{et} \\ q_{ne} & q_{nn} & q_{nu} & q_{nt} \\ q_{ue} & q_{un} & q_{uu} & q_{ut} \\ q_{te} & q_{tn} & q_{tu} & q_{tt} \end{pmatrix} \quad \boldsymbol{H} = \begin{pmatrix} -\boldsymbol{e}_{r,enu}^{s_1}{}^T & 1 \\ -\boldsymbol{e}_{r,enu}^{s_2}{}^T & 1 \\ \vdots & \vdots \\ -\boldsymbol{e}_{r,enu}^{s_m}{}^T & 1 \end{pmatrix}$$

# of satellites = 5



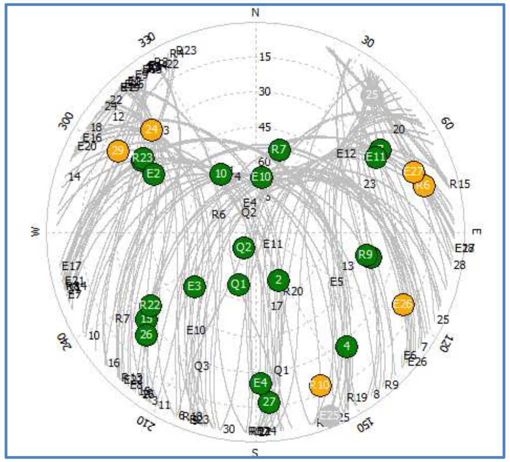
GDOP=33.4 PDOP=25.9
HDOP=8.1 VDOP=24.7

# of satellites = 7



GDOP=2.5 PDOP=2.1
HDOP=1.2 VDOP=1.8

# of satellites = 27



GDOP=1.2 PDOP=1.0
HDOP=0.5 VDOP=0.9

DGPS

- **Differential GPS**
  - Fixed Reference Stations at Known Position
  - Generate Correction Messages
  - Broadcast Correction Messages to User
  - Eliminate Most of Errors of Positioning
- **Service of DGPS**
  - Space Based DGPS: OmniSTAR, SkyFix, StarFix
  - Maritime DGPS: Marine Beacons
  - National DGPS: VHF/FM-band, Cellular Network, Internet

RTCM SC-104

RTCM 2.3 Messages

Type	Message
1	Differential GPS Corrections
3	GPS Reference Station Parameters
10	P-Code Differential Corrections
11	C/A-Code L1, L2 Delta Corrections
17	GPS Ephemerides
18	RTK Uncorrected Carrier Phase
19	RTK Uncorrected Pseudorange
20	RTK Carrier Phase Corrections
21	RTK Pseudorange Corrections
22	Extended Reference Station Parameter
23	Antenna Type Definition Record
24	Antenna Reference Point (ARP)
59	Proprietary Messages

RTCM 3.1 Messages

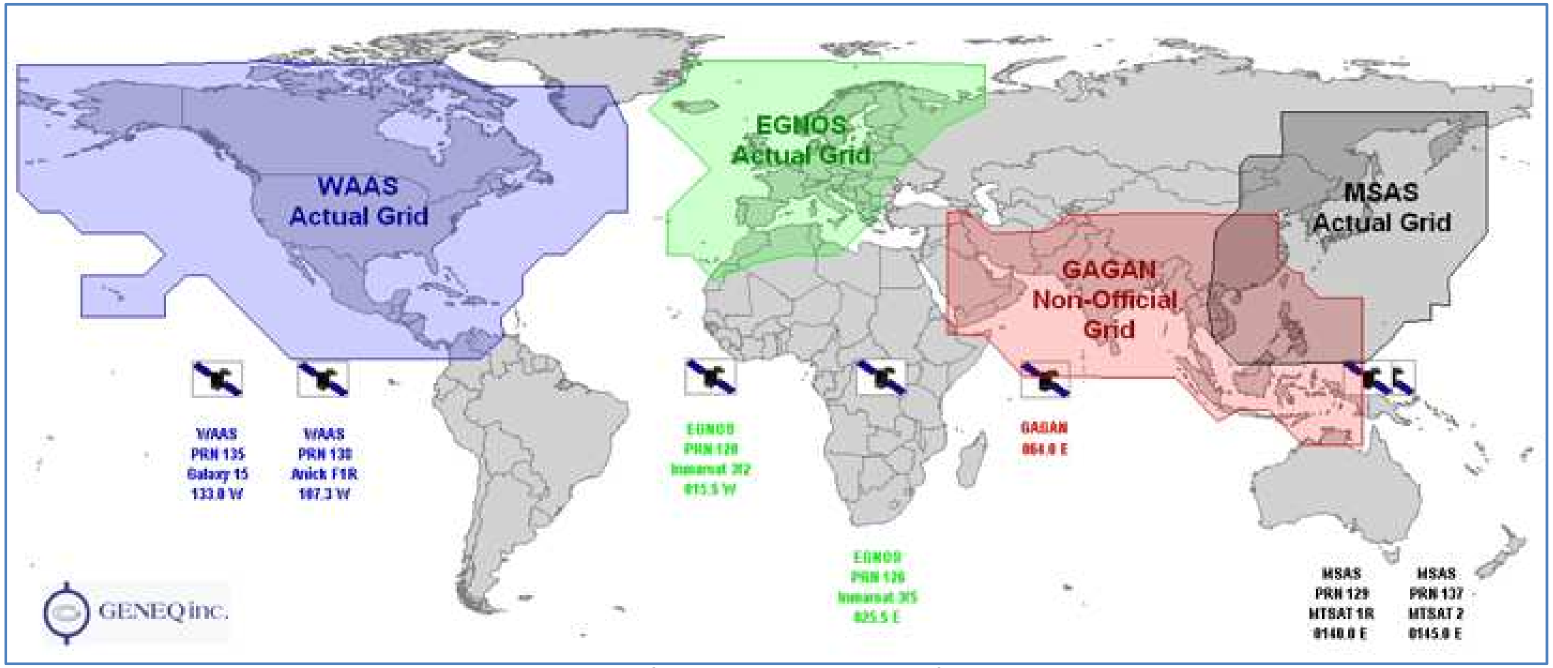
Type	Message
1001	L1-Only GPS RTK Observables
1002	Extended L1-Only GPS RTK Observables
1003	L1&L2 GPS RTK Observables
1004	Extended L1&L2 GPS RTK Observables
1005	Stationary RTK Reference Station ARP
1006	Stationary RTK Ref. Stn. ARP with Hgt.
1007	Antenna Descriptor
1008	Antenna Descriptor & Serial Number
1013	System Parameters
1014	Network Auxiliary Station Data
1015	GPS Ionospheric Correction Differences
1016	GPS Geometric Correction Differences
1019	GPS Ephemerides

RTCM: The Radio Technical Commission for Marine Service

SBAS (Satellite Based Augmentation System)

System	Development	Operation	GEO Satellite		
			PRN	Name	Location
WAAS	US, DOT, FAA	2003/7-	135	Galaxy 15	133W
			138	Anik F1R	107.3W
EGNOS	ESA, EC, Eurocontrol	2009/10,- 2011/3- (SoL)	120	Inmarsat-3 AOR-E	15.5W
			124	Artemis	21.5E
			126	Inmarsat-3 IOR-W	25E
MSAS	Japan, JCAB	2007/9-	129	MTSAT-IR	140E
			137	MTSAT-II	145E
SDCM	Russia	2014-	?	Luch-5A	16E
			?	Luch-5B	95E
			?	Luch-4	167E
GAGAN	India, AAI, ISRO	2011-	127	GSAT-12	?

SBAS Coverage Map



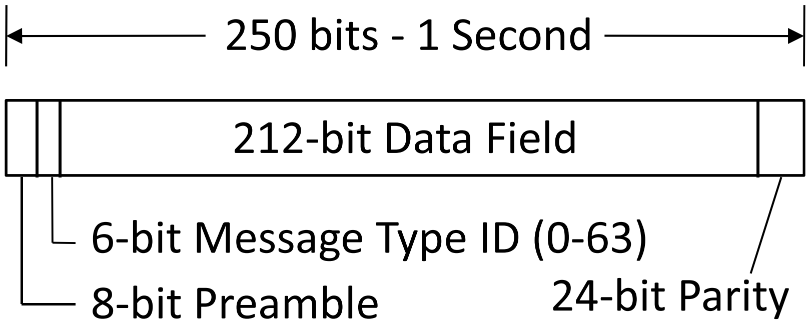
(by GENEQ Inc.)

SBAS Message

RTCA/DO-229C

Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment
(Nov 28,2001)

SBAS Message Frame



RTCA: Radio Technical Commission for Aeronautics

MT	Message
0	For WAAS Testing
1	PRN Mask assignment
2-5	Fast Corrections
6	Integrity Information
7	Fast Correction Degration Factor
9	GEO Navigation Messages
10	Degradation Parameters
12	WAAS Network Time/UTC Offset
17	GEO Satellite Almanac
18	Ionospheric Grid Mask
24	Mixed Fast/Long Term Satellite Correct.
25	Long Term Satellite Error Corrections
26	Ionospheric Delay Corrections
27	WAAS Service Messages

Error Budget

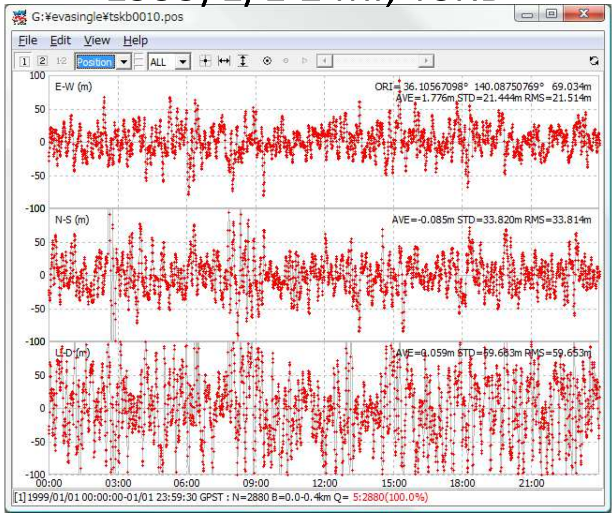
Error Source	Single-Point		DGPS (BL=100km)		SBAS	
Ephemeris Error	1.0 m		0.1 m		0.1 m	
SV Clock Param Error			0.0 m			
Ionospheric Error	1.5 m		0.2 m		0.2 m	
Tropospheric Error	0.3 m		0.1 m		0.3 m	
Multipath	1.0 m		1.2 m		1.0 m	
S/A	0.0 m		0.0 m		0.0 m	
Rcv Tracking Noise	0.3 m		0.3 m		0.3 m	
UERE	2.1 m		1.3 m		1.1 m	
HDOP/VDOP	1.5	2.5	1.5	2.5	1.5	2.5
Horizontal/Vertical RMS Error	**3.2 m**	**5.3 m**	**2.0 m**	**3.3 m**	**1.7 m**	**2.8 m**

Single-Point Positioning

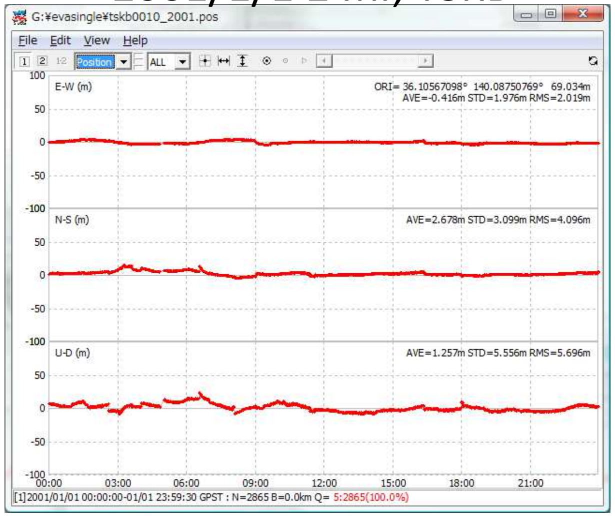
RMS Error:
E: 21.51m
N: 33.81m
U: 59.65m

100m

1999/1/1 24hr, TSKB



2001/1/1 24hr, TSKB



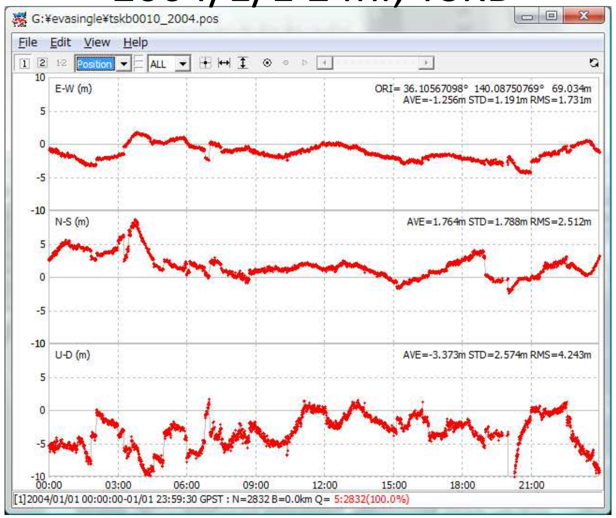
RMS Error:
E: 2.02m
N: 4.10m
U: 5.70m

100m

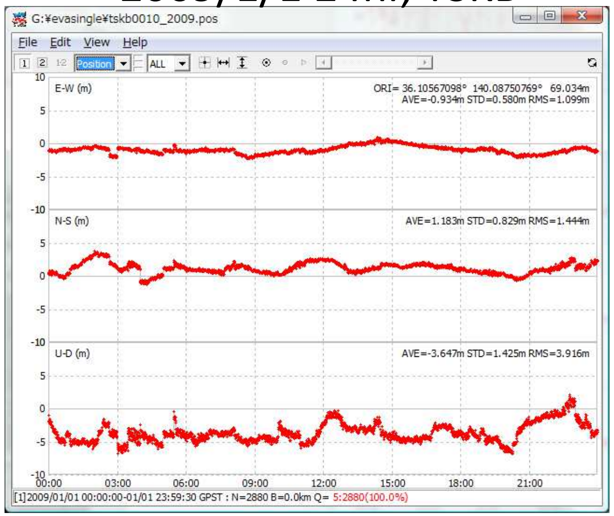
RMS Error:
E: 1.73m
N: 2.51m
U: 4.24m

10m

2004/1/1 24hr, TSKB



2009/1/1 24hr, TSKB



RMS Error:
E: 1.10m
N: 1.44m
U: 3.92m

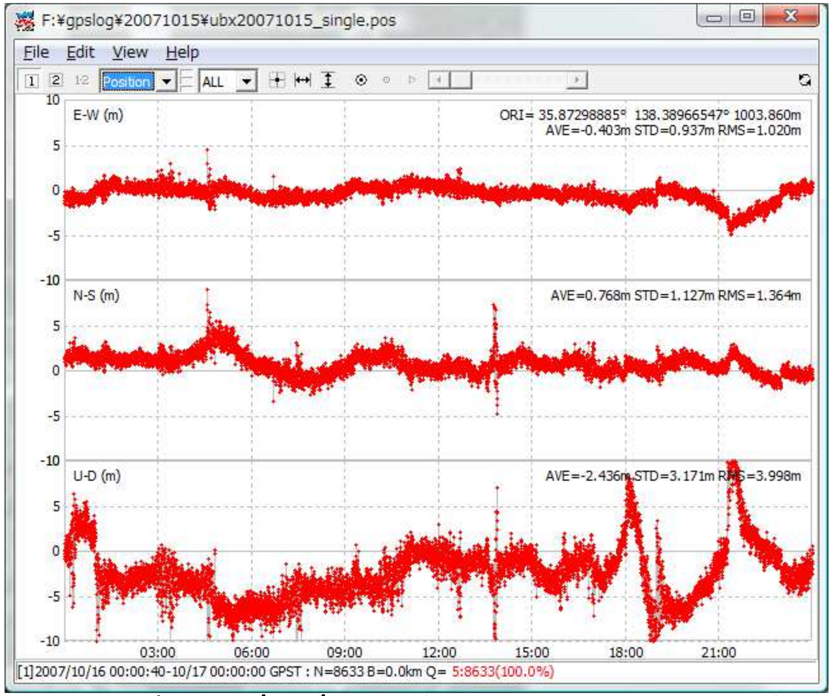
10m

SBAS DGPS Positioning

Single-Point

RMS Error:

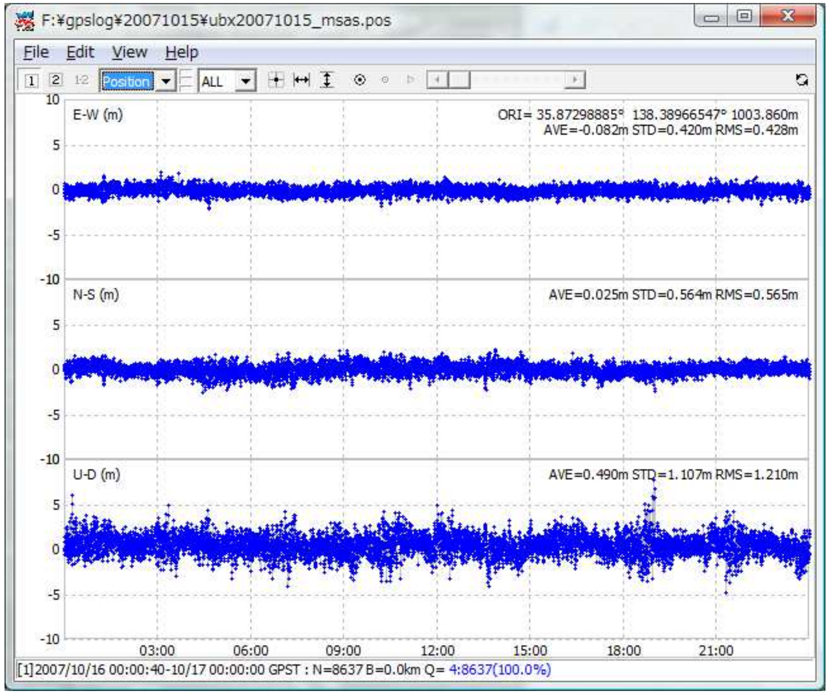
E: 1.02m N: 1.36m U: 4.00m



MSAS

RMS Error:

E: 0.43m N: 0.57m U: 1.21m



(2007/10/16 24hr, Antenna: NovAtel GPS-702-GG, Receiver: u-blox AEK-4T (raw), Processing S/W: RTKLIB 2.1.0, All Corrections=ON, Ranging=ON)

B-4
Theory of Precise Positioning (2)

Code-Based vs. Carrier-Based

	Standard Positioning (code-based)	Precise Positioning (carrier-based)
Observables	Pseudorange (Code)	**Carrier-Phase** + Pseudorange
Receiver Noise	30 cm	3 mm
Multipath	30 cm - 30 m	1 - 3 cm
Sensitivity	High (<20dBHz)	Low (>35dBHz)
Discontinuity	No Slip	Cycle-Slip
Ambiguity	-	Estimated/Resolved
Receiver	Low-Cost (~$100)	Expensive (~$20,000)
Accuracy (RMS)	3 m (H), 5 m (V) (Single) 1 m (H), 2 m (V) (DGPS)	**5 mm (H), 1 cm (V) (Static)** **1 cm (H), 2 cm (V) (RTK)**
Application	Navigation, Timing, SAR,...	Survey, Mapping, ...

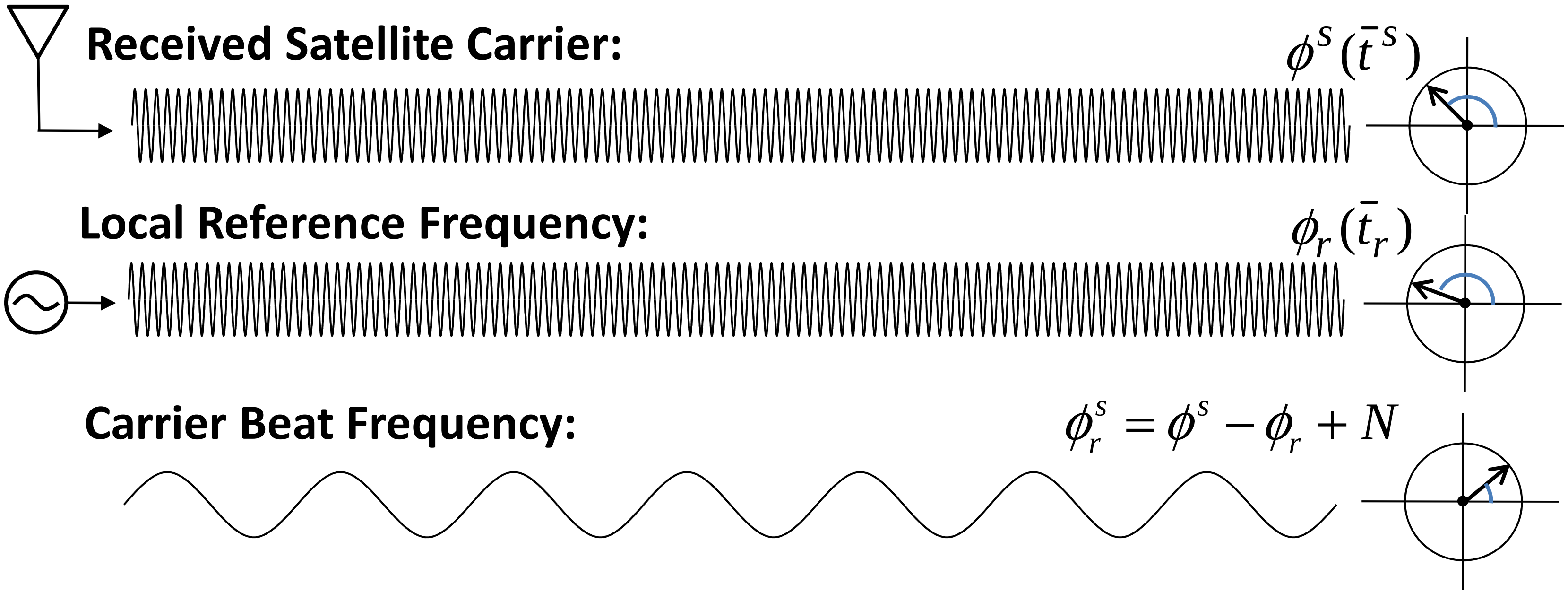
Carrier-Phase

Definition:

$$\phi_r^s = \phi^s - \phi_r + N$$

(cycle)

... actually being a measurement on the beat frequency between the received carrier of the satellite signal and a receiver-generated reference frequency. *(RINEX 2.10)*



Carrier-Phase Model (1)

Carrier-Phase:

$$\phi_r^s = \phi_r(t_r) - \phi^s(t^s) + N_r^s + \varepsilon_\varphi \qquad (\phi_{r,0} = \phi_r(t_0), \phi_0^S = \phi^S(t_0))$$

$$= (f(t_r + dt_r - t_0) + \phi_{r,0}) - (f(t^s + dT^s - t_0) + \phi_0^s) + N_r^s + \varepsilon_\varphi$$

$$= \frac{c}{\lambda}(t_r - t^s) + \frac{c}{\lambda}(dt_r - dT^s) + (\phi_{r,0} - \phi_0^s + N_r^s) + \varepsilon_\varphi \qquad \text{(cycle)}$$

$$\Phi_r^s \equiv \lambda\phi_r^s = c(t_r - t^s) + c(dt_r - dT^s) + \lambda(\phi_{r,0} - \phi_0^s + N_r^s) + \lambda\varepsilon_\varphi$$

$$= \underline{\rho_r^s + c(dt_r - dT^s) - I_r^s + T_r^s} + \lambda \underline{B_r^s} + \underline{d_r^s} + \varepsilon_\Phi \qquad \text{(m)}$$

Carrier-Phase Bias Other Correction Terms

Pseudorange:

$$P_r^s = \underline{\rho_r^s + c(dt_r - dT^s) + I_r^s + T_r^s} + \varepsilon_P$$

Carrier-Phase Model (2)

Carrier-Phase Bias:

$$B_r^s = \phi_{r,0} - \phi_0^s + N_r^s \qquad \text{(cycle)}$$

N_r^s : **Integer Ambiguity**

$\phi_{r,0}$: Receiver Initial Phase

ϕ_0^s : Satellite Initial Phase

Other Correction Terms:

$$d_r^s = -\boldsymbol{d}_{r,pco}{}^T \boldsymbol{e}_{r,enu}^s + \left(\boldsymbol{E}_{sat\rightarrow ecef}\,\boldsymbol{d}_{pco}^s\right)^T \boldsymbol{e}_r^s + d_{r,pcv} + d_{pcv}^s - \boldsymbol{d}_{disp}{}^T \boldsymbol{e}_{r,enu}^s + d_{pw} + d_{rel} \qquad \text{(m)}$$

$\boldsymbol{d}_{r,pco}$: Receiver Antenna Phase Center Offset

$d_{r,pcv}$: Receiver Antenna Phase Center Variation

$\boldsymbol{d}_{pco}^s$: Satellite Antenna Phase Center Offset

d_{pcv}^s : Satellite Antenna Phase Center Variation

$\boldsymbol{d}_{disp}$: Site Displacement

d_{pw} : Phase Wind-up Effect

d_{rel} : Relativistic Effect

DD (Double Difference)

$$\Phi_{ub}^{ij} \equiv \lambda((\phi_u^i - \phi_b^i) - (\phi_u^j - \phi_b^j))$$
$$= \rho_{ub}^{ij} + c(dt_{ub}^{ij} - dT_{ub}^{ij}) - I_{ub}^{ij} + T_{ub}^{ij} + \lambda B_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_\Phi$$
$$= \rho_{ub}^{ij} - I_{ub}^{ij} + T_{ub}^{ij} + \lambda N_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_\Phi$$

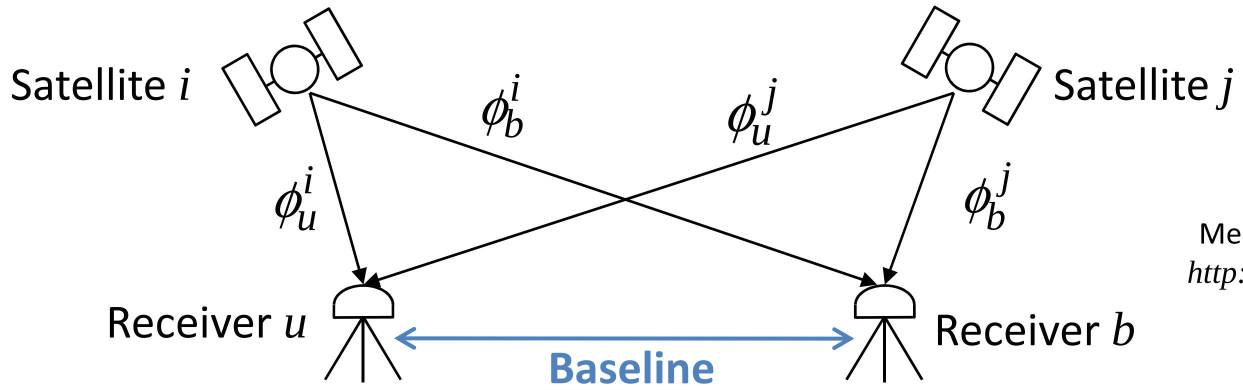
$$dt_{ub}^{ij} = dt_u^{ij} - dt_b^{ij} = 0,\ dT_{ub}^{ij} = dT_{ub}^i - dT_{ub}^j \approx 0$$

$$B_{ub}^{ij} = (\phi_{u,0} - \phi_0^i + N_u^i) - (\phi_{b,0} - \phi_0^i + N_b^i) - (\phi_{u,0} - \phi_0^j + N_u^j) + (\phi_{b,0} - \phi_0^j + N_b^j) = N_{ub}^{ij}$$

(short Baseline and same antenna type)

$$\Phi_{ub}^{ij} \approx \rho_{ub}^{ij} + \lambda N_{ub}^{ij} + \varepsilon_\Phi$$

$$I_{ub}^{ij} = I_{ub}^i - I_{ub}^j \approx 0, T_{ub}^{ij} = T_{ub}^i - T_{ub}^j \approx 0, d_{ub}^{ij} = d_{ub}^i - d_{ub}^j \approx 0$$



Memo for Misra & Enge:
http://gpspp.sakura.ne.jp/diary200608.htm

Carrier-based Relative Positioning

Nonlinear-LSE:

Parameter Vector:

$$\boldsymbol{x} = (\boldsymbol{r}_u{}^T, N_{ub}^{s_2 s_1}, N_{ub}^{s_3 s_1}, \ldots, N_{ub}^{s_m s_1})^T$$

Measurement Vector:

$$\boldsymbol{y} = (\boldsymbol{y}_{t_1}{}^T, \boldsymbol{y}_{t_1}{}^T, \ldots, \boldsymbol{y}_{t_n}{}^T)^T$$

Meas Model, Design Matrix:

$$\boldsymbol{h}(\boldsymbol{x}) = \left(\boldsymbol{h}_{t_1}(\boldsymbol{x})^T, \boldsymbol{h}_{t_2}(\boldsymbol{x})^T, \ldots, \boldsymbol{h}_{t_n}(\boldsymbol{x})^T\right)^T$$

$$\boldsymbol{H} = \left(\boldsymbol{H}_{t_1}{}^T, \boldsymbol{H}_{t_2}{}^T, \ldots, \boldsymbol{H}_{t_n}{}^T\right)^T$$

Meas Error Covariance:

$$\boldsymbol{R} = blkdiag\left(\boldsymbol{R}_{t_1}, \boldsymbol{R}_{t_2}, \ldots, \boldsymbol{R}_{t_n}\right)$$

Solution (Static/Float):

$$\hat{\boldsymbol{x}} = \boldsymbol{x}_0 + (\boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{H})^{-1} \boldsymbol{H}^T \boldsymbol{R}^{-1} (\boldsymbol{y} - \boldsymbol{h}(\boldsymbol{x}_0))$$

$$\boldsymbol{y}_{t_k} = (\Phi_{ub,t_k}^{s_2 s_1}, \Phi_{ub,t_k}^{s_3 s_1}, \ldots, \Phi_{ub,t_k}^{s_m s_1})^T$$

$$\boldsymbol{h}_{t_k}(\boldsymbol{x}) = \begin{pmatrix} \rho_{u,t_k}^{s_2 s_1} - \rho_{b,t_k}^{s_2 s_1} + \lambda N_{ub}^{s_2 s_1} \\ \rho_{u,t_k}^{s_3 s_1} - \rho_{b,t_k}^{s_3 s_1} + \lambda N_{ub}^{s_m s_1} \\ \vdots \\ \rho_{u,t_k}^{s_m s_1} - \rho_{b,t_k}^{s_m s_1} + \lambda N_{ub}^{s_m s_1} \end{pmatrix}$$

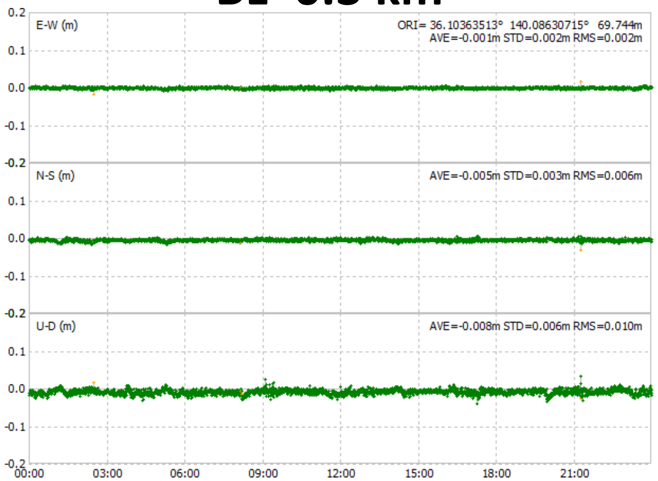
$$\boldsymbol{H}_{t_k} = \begin{pmatrix} -\boldsymbol{e}_{u,t_k}^{s_2 s_1 T} & \lambda & 0 & \cdots & 0 \\ -\boldsymbol{e}_{u,t_k}^{s_3 s_1 T} & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{e}_{u,t_k}^{s_m s_1 T} & 0 & 0 & \cdots & \lambda \end{pmatrix}$$

$$\boldsymbol{R}_{t_k} = \begin{pmatrix} 4\sigma_\Phi{}^2 & 2\sigma_\Phi{}^2 & \cdots & 2\sigma_\Phi{}^2 \\ 2\sigma_\Phi{}^2 & 4\sigma_\Phi{}^2 & \cdots & 2\sigma_\Phi{}^2 \\ \vdots & \vdots & \ddots & \vdots \\ 2\sigma_\Phi{}^2 & 2\sigma_\Phi{}^2 & \cdots & 4\sigma_\Phi{}^2 \end{pmatrix}$$

r_b : Fixed Base-Station Position

Effect of Baseline Length

BL=0.3 km

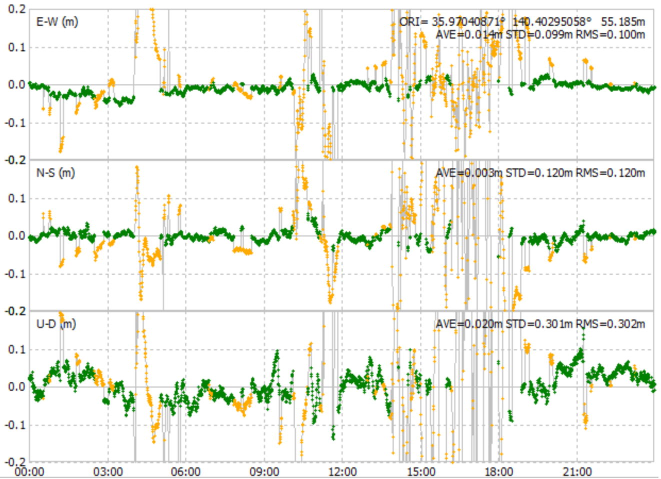


RMS Error:
E: 0.2cm
N: 0.6cm
U: 1.0cm
Fix Ratio:
99.9%

BL=13.3 km

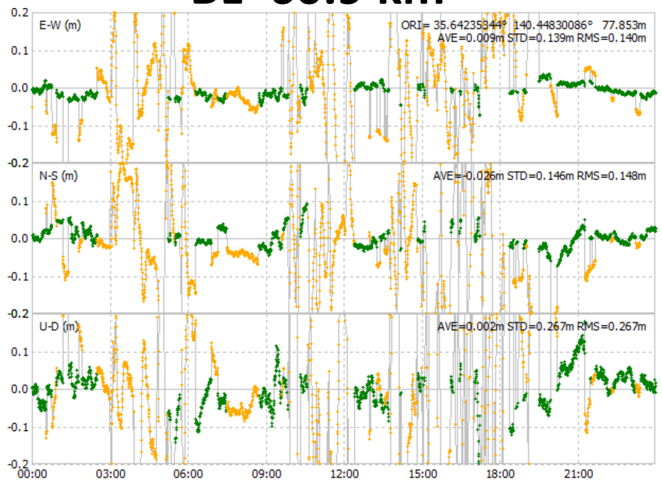
RMS Error:
E: 2.2cm
N: 2.4cm
U: 10.6cm
Fix Ratio:
94.2%

BL=32.2 km



RMS Error:
E: 10.0cm
N: 12.0cm
U: 30.2cm
Fix Ratio:
64.3%

BL=60.9 km



RMS Error:
E: 14.0cm
N: 14.8cm
U: 26.7cm
Fix Ratio:
44.4%

(24 hr Kinematic ●: Fixed Solution ●: Float Solution)

Integer Ambiguity Resolution

- **Objectives**
  - More accurate than float solutions
  - Fast converge of solutions
- **Many AR Strategies**
  - Simple Integer rounding
  - Multi-frequency wide-lane and narrow-lane generation
  - Search in coordinate domain
  - Search in ambiguity domain
  - AFM, FARA, LSAST, LAMBDA, ARCE, HB-L^3, Modified Cholesy Decomposition, Null Space, FAST, OMEGA, ...

ILS (Integer Least Square Estimation)

Problem:

$$x = (a^T, b^T)^T,\; H = (A, B)$$

$$y = Hx + v = Aa + Bb + v$$

$$\breve{x} = \underset{a \in Z^n, b \in R^m}{\arg\min}\; (y - Hx)^T Q_y^{-1} (y - Hx)$$

Strategy:

(1) Conventional LSE

$$\hat{x} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = Q_x H^T Q_y^{-1} y,\; Q_x = \begin{pmatrix} Q_a & Q_{ab} \\ Q_{ba} & Q_b \end{pmatrix} = (H^T Q_y H)^{-1}$$

(2) **Search Integer Vector** with Minimum Squared Residuals

$$\breve{a} = \underset{a \in Z^n}{\arg\min} (\hat{a} - a)^T Q_a^{-1} (\hat{a} - a)$$

(3) Improve solution

$$\breve{b} = \hat{b} - Q_{ba} Q_a^{-1} (\hat{a} - \breve{a})$$

LAMBDA

Teunissen, P.J.G. (1995)
The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. *Journal of Geodesy,* Vol. 70, No. 1-2, pp. 65-82.

- **ILS Estimation with**:
  - Shrink Integer Search Space with "Decorrelation"
  - Efficient Tree Search Strategy
  - Similar to *Closest Point Search with LLL Lattice Basis Reduction* Algorithm

$$\breve{\boldsymbol{a}} = \arg\min_{\boldsymbol{a} \in \boldsymbol{Z}^n} (\hat{\boldsymbol{a}} - \boldsymbol{a})^T \boldsymbol{Q}_a{}^{-1} (\hat{\boldsymbol{a}} - \boldsymbol{a})$$

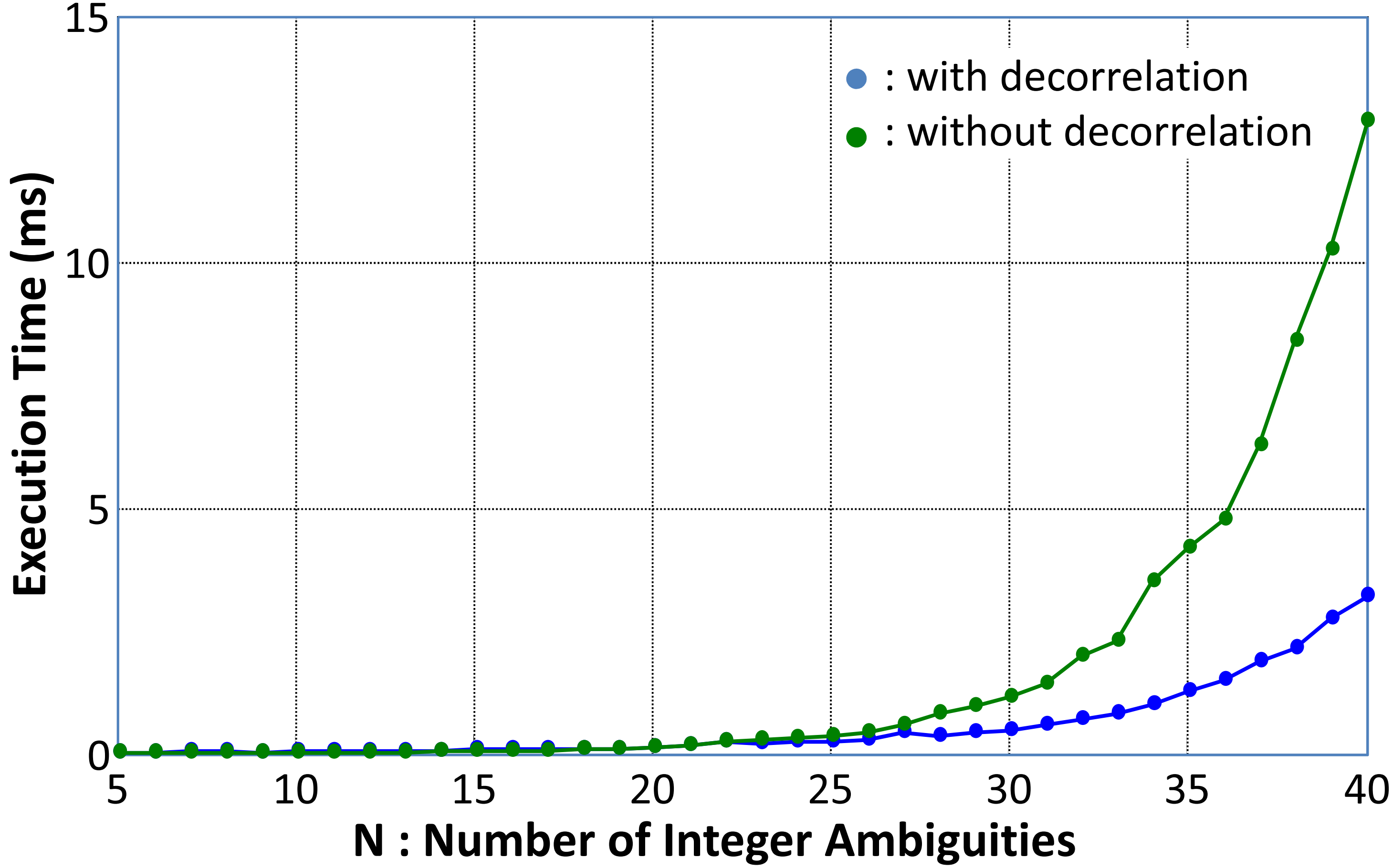
⇔

$$\hat{\boldsymbol{z}} = \boldsymbol{Z}^T \hat{\mathbf{a}},\ \mathbf{Q}_z = \boldsymbol{Z}^T \mathbf{Q}_a \boldsymbol{Z}$$

$$\breve{\boldsymbol{z}} = \arg\min_{\boldsymbol{z} \in \boldsymbol{Z}^n} (\hat{\boldsymbol{z}} - \boldsymbol{z})^T \boldsymbol{Q}_z{}^{-1} (\hat{\boldsymbol{z}} - \boldsymbol{z})$$

$$\breve{\boldsymbol{a}} = \boldsymbol{Z}^{-T} \breve{\boldsymbol{z}}$$

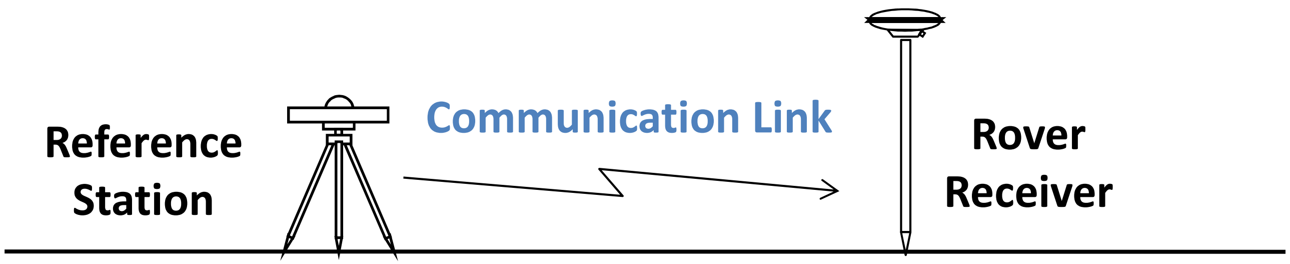
Performance of LAMBDA



(Pentium 4 3.2GHz, Intel C/C++ 8.0)

RTK (Real-time Kinematic)

- **Technique with Carrier-based Relative Positioning**
  - Real-time Position of Rover Antenna
  - Transmit Reference Station Data to Rover via Comm. Link
  - **OTF** (On-the-Fly) Integer Ambiguity Resolution
  - Typical Accuracy: 1 cm + 1ppm x BL RMS (Horizontal)
  - Applications:
    Land Survey, Construction Machine Control, Precision Agriculture etc.



RTK Application

Geodetic Survey

Construction Machine Control

Precision Agriculture

ITS (Intelligent Transport System)

Mobile Mapping System

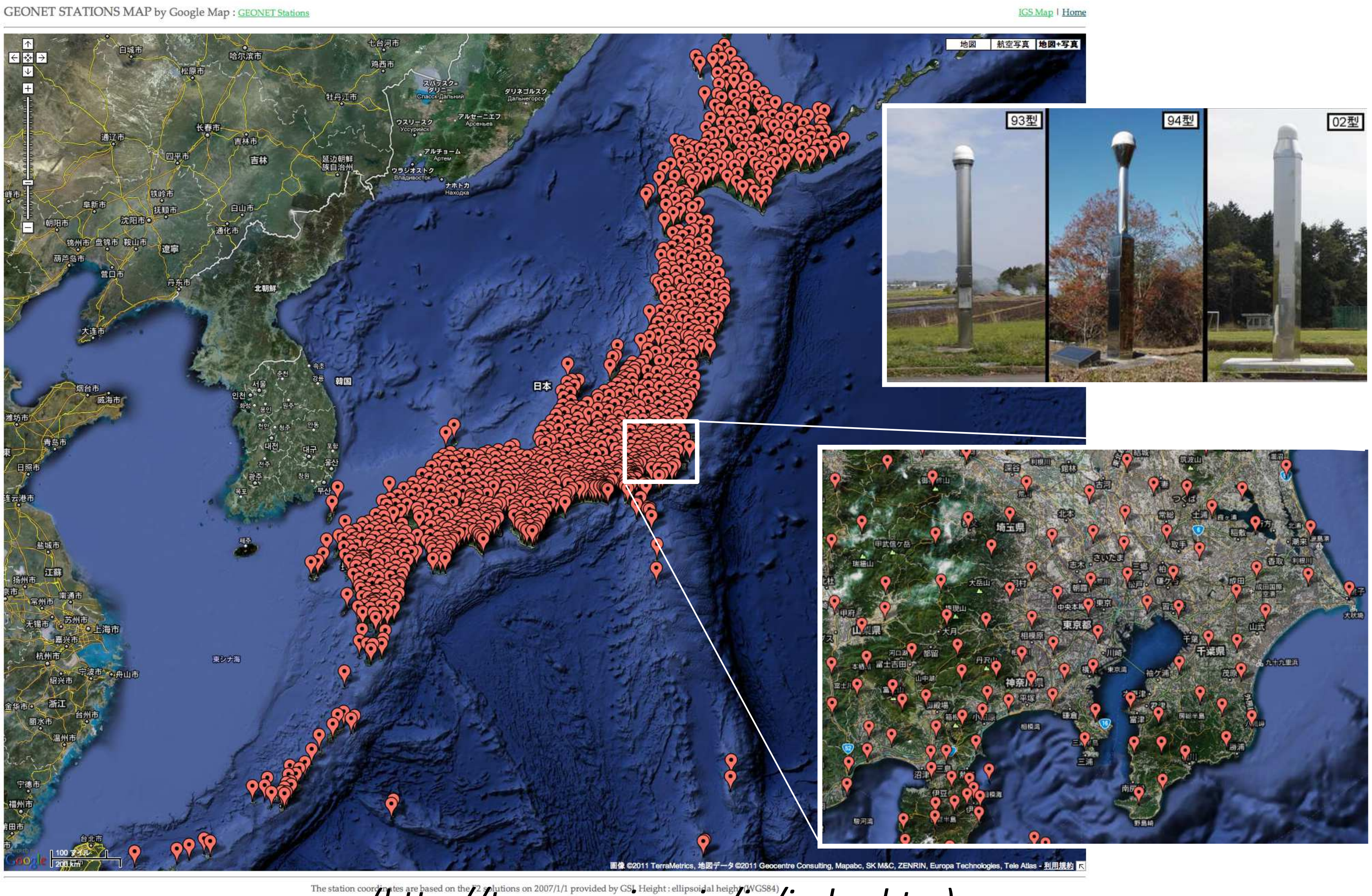
Sports

http://www.trimble.com, http://www.leica-geosystems.com, http://www.gpsworld.com

NRTK (Network RTK)

- **Extension of RTK**
  - RTK without User Reference Station
  - Sparse Networked Reference Stations
  - Correction Messages via Mobile-Phone Network
  - Format: **VRS**, **FKP**, MAC, RTCM 2.3, RTCM 3.1
  - Server S/W: Trimble GPSNet, GEO++ GNSMART, ...
  - NTRIP Networked Transport of RTCM via Internet Protocol
- **NRTK Service in Japan**
  - GEONET: ~1200 Reference Stations by GSI
  - NGDS (www.gpsdata.co.jp), JENOBA (www.jenoba.jp)

Japanese GEONET



(http://terras.gsi.go.jp/ja/index.htm)

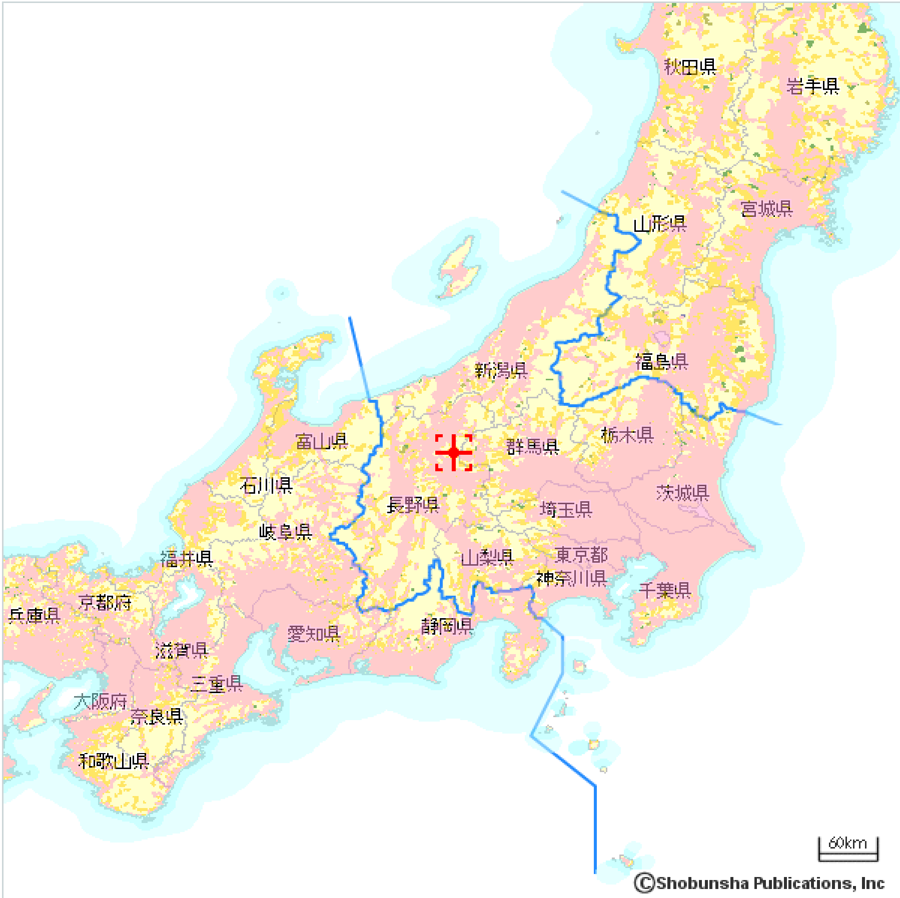
Considerations for RTK System

- **Rover**
  - Single vs. Dual-freq, Update Rate, GNSS, Receiver-cost
  - CPU Power for external processing
  - INS-integration for obstacles
- **Reference Station**
  - Baseline-Length vs. Performance
  - Self-provided vs. NRTK Service
  - Coverage, Receiver-cost, Operational-cost, Service-fee
- **Communication Link**
  - Coverage, Band-width, Latency, Link-cost

Communication Link for RTK

- **Local (<300 m)**
  - Serial, USB, LAN, ... (wired)
  - Radio Modem, WiFi, ZigBee, DSRC, ... (wireless)
- **Regional (<1,000 km)**
  - Analog-phone, ISDN, Dedicated Link, ... (wired)
  - Mobile-phone (Analog, 2G, 3G, …), ... (wireless)
- **Global (<10,000 km)**
  - Internet
  - GEO Satellite Link (Inmarsat, WideStar II, ...)
  - LEO Satellite Link (Iridum, Orbicom, …)

Coverage by Mobile-phone N/W



NTT docomo FOMA (2008/9)

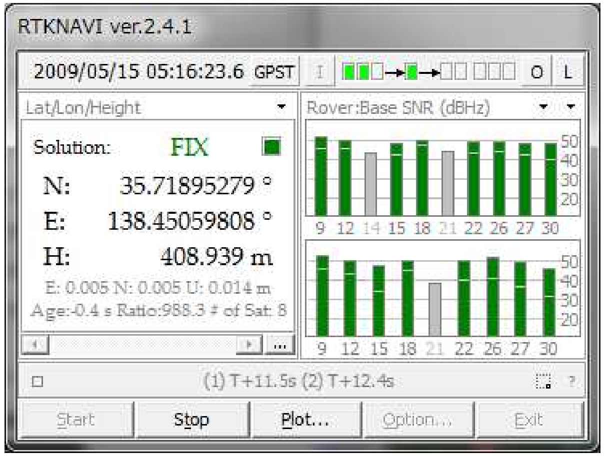
(http://servicearea.nttdocomo.co.jp)

B-5
Practice of RTK

RTK by Playback Data

- **Objective**
  RTK of by Playback Data
- **Program**
  rtklib_2.4.2¥bin¥rtknavi.exe
- **Data**
  sample2¥
  oemv_2009515c.gps (NovAtel)
  0263_20090515c.rtcm3 (VRS)

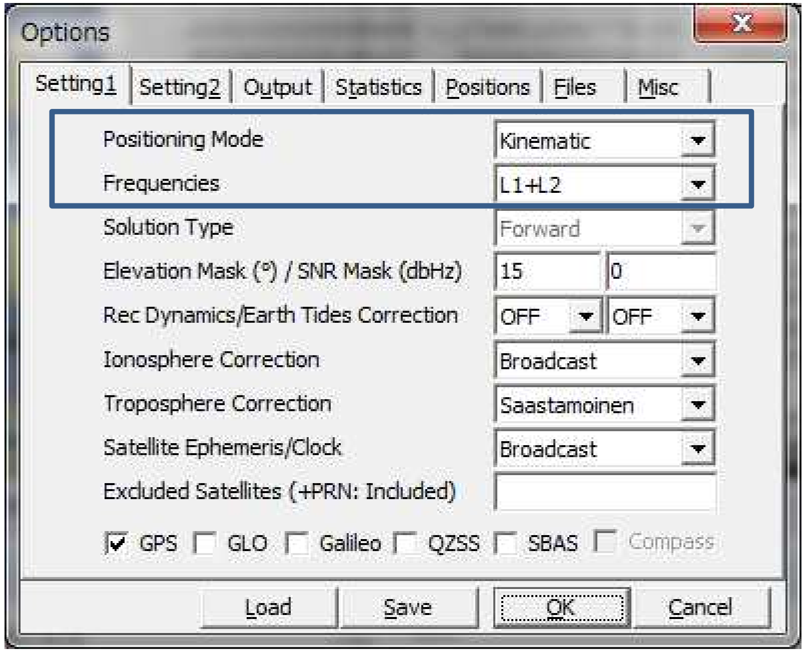
RTKNAVI



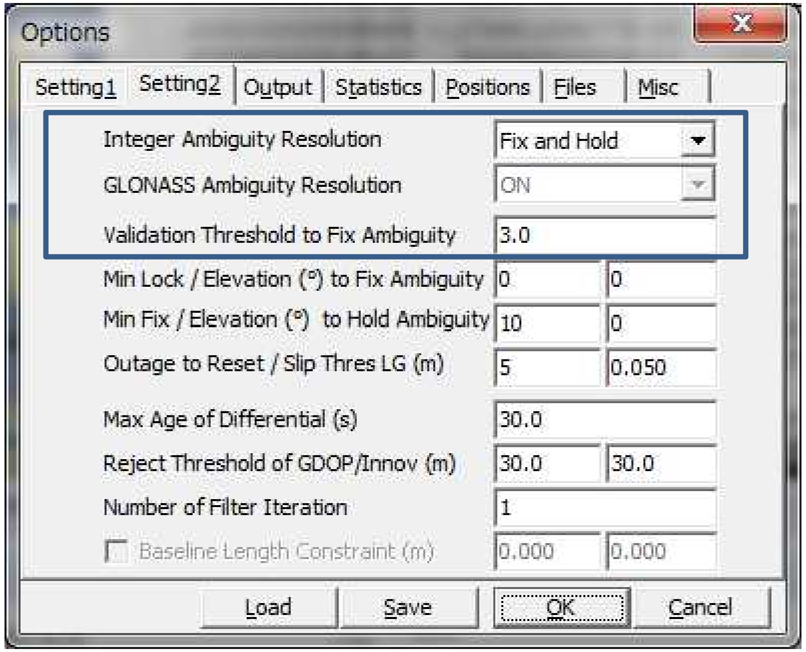
NovAtel OEMV-3G,
GPS-702-GG

RTKNAVI - Options

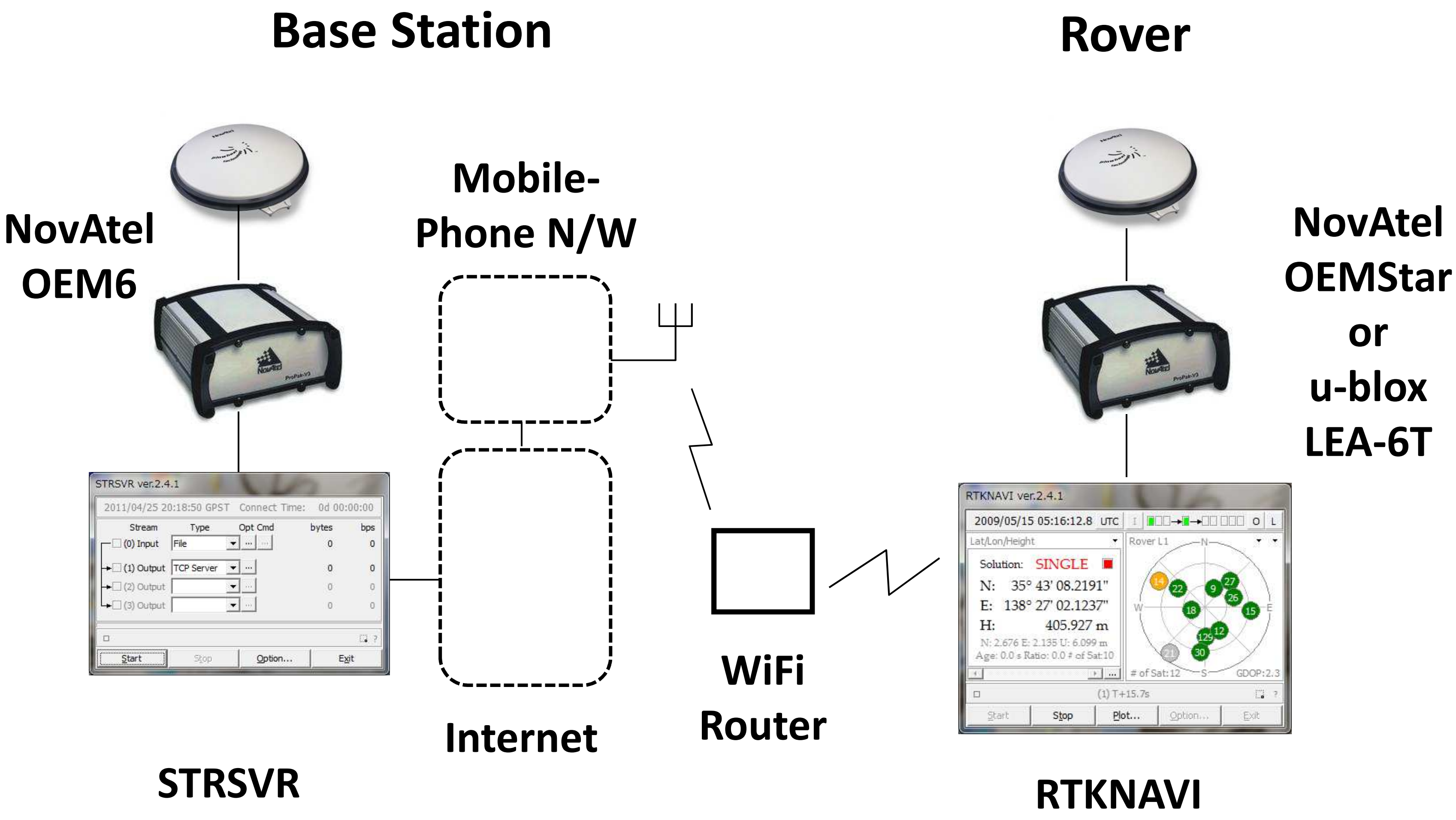
Setting1



Setting2



RTK Configuration

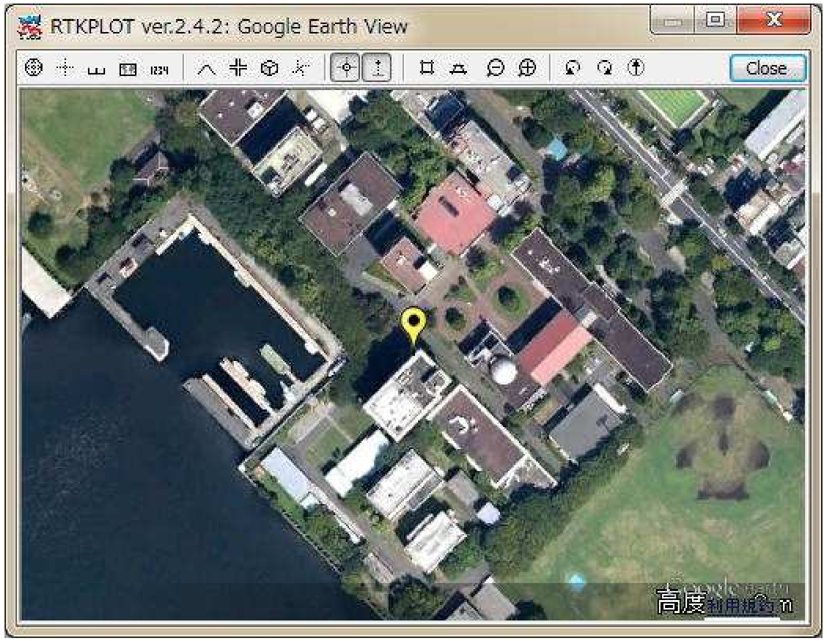


Receiver Setup (NovAtel OEMStar)

- **Install FTDI USB-Serial Driver (if necessary)**
  CDM20814_WHQL_Certified
- **Install NovAtel Connect Utility**
  NovAtelConnect_1.4.0.exe
- **Execute NovAtel Connect**
- **Check OEMStar – PC connection**

Base Station Data

- NTRIP Caster: *********, Port: **2101** or **80**
- Mountpoint: **TUN0**
- User-ID: ******, Password: ******
- Format: **NovAtel OEM6**
- Antenna: **JPSLEGANT_E**
- Latitude: **35.666496999**
- Longitude: **139.792372076**
- Height: **59.4046**



Practice of RTK

- Setup **RTKNAVI** input streams
  - Rover Stream: serial connected to PC
  - Base Station Stream: NTRIP via Internet
- Setup **RTKNAVI** options
  - RTK options Setup
  - Ambiguity resolution options
  - Output options
  - Base station position

Receiver Setup (u-blox LEA-6T)

- **Install u-blox Windows Driver**
  ublox_A4_U5_USB_drv3264_install_UI.exe
- **Install u-blox u-center utility**
  u-centerSetup-7.0.2.1.exe
- **Execute u-blox u-center**
- **Check u-blox - PC connection**