

# TEC Measurement By Single Dual-frequency GPS Receiver

Yun Zhang, Falin Wu, Nobuaki Kubo and Akio Yasuda  
Tokyo University of Mercantile Marine  
(2-1-6 Etchujima, Koto-ku, Tokyo 135-8533 Japan)

## BIOGRAPHY

Yun Zhang is a master student at the Tokyo University of Mercantile Marine (TUMM). He received his B.Sc degree from Shanghai Maritime University, P.R. China, in 1996. He worked at Shanghai China Ocean Company (COSCO) from 1996 to 2001. His current research interest is estimating the Total Electron Content by GPS receiver.

Falin Wu is a Ph.D. student in the laboratory of communication engineering at TUMM. He received his B.Sc and M.Sc degrees from Dalian Maritime University, P.R. China, in 1995 and in 1998 respectively. His current research interest is RTK positioning.

Nobuaki Kubo is currently a research associate at TUMM. He graduated from Hokkaido University in 1996 with a Bachelor of Engineering (Electrical) and with a Master of Engineering (Electrical) in 1998. He joined NEC Company in 1998 and has engaged in GBAS system for 3 years. He is interested in multi-path estimating in GPS.

Akio Yasuda is a professor at TUMM. He received his Dr. Eng. Degree from Nagoya University, Japan. He is the president of GPS Society,

## ABSTRACT

By GPS dual-frequency receiver, the Total Electron Content (TEC) along the path from satellite to receiver can be measured directly. But the dual-frequency measurement can be severely colored by the thermal noise in the GPS receiver and the bias in the GPS satellite. The new two steps method is proposed to estimate the precise quantities of TEC.

In the first step, the "phase leveling" results of the ionospheric delay would be measured. By filtering the carrier and code measurement together, the "post-processing measurement" is used to average the code measurements from carrier phase track

to calibrate the relative carrier ambiguity, but the bias of the GPS receiver and satellites is remained in this step. The remained bias is called inter-frequency bias (IFB).

In the second step the SLM (single layer model) assumption and the least squares are used to determine IFB. Through one-minute average of the results measured in the first step, one-minute average of vertical TEC (VTEC) can be measured and IFB can be calibrated. Setting two GPS receivers through one antenna tests this proposed method, one minute average of VTEC with two receivers are measured respectively; the difference between VTEC measured by receiver1 and receiver2 are calculated and the value of the difference are very little. It shows that the method can calibrate IFB, and also can measure the one-minute average of vertical TEC by single GPS observer.

## INTRODUCTION

The ionosphere causes GPS signal delays due to the TEC along the path from the GPS satellite to receiver. Under normal solar activity conditions, the influence on GPS signal by TEC are usually in the range from a few to tens meters [1]. For high precision GPS positioning, the ionosphere effect must be estimated so that the more precise position result could be measured.

The TEC by dual-frequency measurement includes IFB, if this bias is not calibrated; the result is not the absolute TEC what we require. To measure precise vertical TEC by single GPS receiver, a new method is proposed in this paper. To reduce the pseudorange error and measure changes of the ionospheric delay correctly, the post-processed measurement is used to calibrate the relative cycle ambiguity with dual-frequency GPS receiver and estimate the "phase-leveling" result [2], because the group delay is irregularity but absolute and the carrier phase advance is relative range error in the ionosphere. By using the SLM assumption [3], the slant TEC (STEC) can be converted into VTEC of certificated height, and then, IFB could be

measured with two satellites of the highest and the second highest elevation by the weighted least square, and also, one-minute average vertical TEC would be estimated.

Finally the experiment was conducted from 21:00 April 7 to 07:00 April 8 2003 (Local Time) for 10 hours to examine the valid of the new method, we calculated the difference between VTEC with two receivers and compared the VTEC measured by receiver1 with the results of IRI-95 model [4].

## IONOSPHERE MEASUREMENT BY GPS RECEIVER

With the first-order refraction index, the ionospheric delay would be measured by dual-frequency GPS receiver. Because the higher noise level of the ionosphere measurement by code measurement, It called as “post-processed ionosphere measurement” is used to estimate the “phase-leveling” results of ionospheric delay.

### Total Electron Content

The ionosphere is a region of ionized gases (free electron and ions). The ionization is caused by the sun’s radiation, and the state of the ionosphere is determined primarily by the intensity of the solar activity. The parameter of the ionosphere that produces most of the effects on GPS signals is the total number of electrons in the ionosphere. This integrated number of electrons, commonly called TEC. A value of TEC equal to  $1 \times 10^{16} \text{ electrons} / \text{m}^2$  is called one TEC unit (TECU).

TEC typically varies 1 and 150 TECU. And TEC is determined as following [5]:

$$TEC = \int_s^R N dl \quad (1)$$

where  $R$  is receiver,  $s$  is satellite,  $N$  is the local electron density, and the integration is along the signal path from the satellite to the receiver. The path length through the shortest in the zenith direction and, therefore,

### Group Delay

The ionosphere is an important source of range and range-rate errors for GPS users, and the variability of the ionosphere is much larger than troposphere. Fortunately, ionized gas is a dispersive medium for radio waves [8]. The refraction index is a function of the operating frequency. The group delay of the ionosphere produces range errors, which can be expressed into

the units of distance to GPS users. From the expression for the refractive index, the group delay can be determined by following:

$$\Delta r = \int (1 - n) dl \quad (2)$$

where  $n$  is the refractive index of the ionosphere, the integration is along the signal path from the satellite to the receiver.

### Refraction index

To quantify the propagation effects on a radio wave traveling through the ionosphere, the refractive index of the medium must be specified, the refractive index of the ionosphere,  $n$ , has been derived by Appleton and Hartree [6]. Because we are concerned here only with radio-wave propagation at two GPS frequencies ( $L1=1575\text{MHz}$ ,  $L2=1227\text{MHz}$ ), the refractive index of the ionosphere at GPS frequencies given by Brunner and Gu [7], can be expressed as

$$n = 1 - \left(\frac{X}{2}\right) \pm \left(\frac{1}{f^3}\right) - \left(\frac{1}{f^4}\right) \quad (3)$$

where  $f$  is system operation frequency, and,

$$X = f_n^2 / f^2, f_n \text{ is the ionospheric plasma frequency.}$$

As Eq.(3) illustrates, the terms contributing to the refractive index of the ionosphere are the free-space velocity, the first-order term (as often used in GPS TEC measurement), the second-order term and the third order term. The refractive index of the first order in the ionosphere is given by the following:

$$n = 1 - \left(\frac{X}{2}\right) \quad (4)$$

By Eq.(3) and Eq.(4), the high-order terms are much less than 1% of the first-order terms at GPS frequencies, so with better than 0.1% accuracy, even during worst case ionosphere conditions when  $f_n = 25\text{MHz}$  [8], so the ionospheric refractive index at GPS frequency can be expressed simply as first-order term.

Now using Eq.(4) the first-order refractive index, the refraction index for a radio wave of frequency is:

$$n \approx 1 - \frac{40.3N}{f^2} \quad (5)$$

where  $N$  is local iono density,  $f$  is system operation frequency,.

By Eq.(1) and Eq.(2), the ionospheric group delay is

$$\Delta r = \frac{40.3}{f^2} \int Ndl = \frac{40.3}{f^2} TEC \quad (6)$$

So it can be calculated that TEC is effected range delay into meters at  $f_{L1}$  of the GPS: 1TECU=0.163m; and effected on  $f_{L2}$ : 1TECU=0.267m;

In this paper, The TECU would be converted into unit of distance at L1 GPS frequency to estimate vertical TEC.

### Post-processing Ionosphere Measurements

By dual-frequency GPS receiver, we could measure the ionospheric delay, however, the results are including IFB [9].

$$I^t_i = STEC^t + M + IFB_i \quad (7)$$

where  $t$  is the epoch time,  $i$  is PRN of GPS satellite,  $M$  is multi-path,  $I$  is the result by dual-frequency ionosphere measurement;  $STEC$  is the slant TEC measurement;  $M$  is multi-path influence,  $IFB$  is the inter-frequency bias including the receiver and the satellite biases. IFB is very stable over time on a scale of days to months in practice so that it can be treated as constant in our measurement [11].

The group delay and ionospheric carrier phase advance with the dual frequency GPS receiver can be measured simply under the first-order refractive index [12], and the ionospheric delay along the path from satellite to receiver would be estimated directly as following [1]:

Code measurement:

$$STEC = I_r - M_r - IFB \quad (8)$$

$$\text{where } I_r = \mathbf{g}(\mathbf{r}_{L1} - \mathbf{r}_{L2}) \quad (9)$$

Carrier phase measurement:

$$STEC = I_f - M_f - N_{12} - IFB \quad (10)$$

$$\text{where } I_f = -\mathbf{g}(\mathbf{f}_{L1} - \mathbf{f}_{L2}) \quad (11)$$

Here,  $= f_{L2}^2 / (f_{L2}^2 - f_{L1}^2)$  converts the TEC into meters at L1GPS frequency

$I_r$  is the ionospheric group delay (m),  $I_f$  is ionospheric

carrier phase advanced (m),  $\mathbf{r}_{L1}$  and  $\mathbf{r}_{L2}$  are pseudorange measured by code measurement at L1 and L2 frequency (m),  $\mathbf{f}_{L1}$  and  $\mathbf{f}_{L2}$  are pseudorange measured by carrier phase measurement at L1 and L2 frequency (m),  $N_{12}$  is the relative cycle ambiguity of the carrier phased measurement (m),  $M_r$  and  $M_f$  are multi-path by code measurement (m) and carrier phase measurement (m)

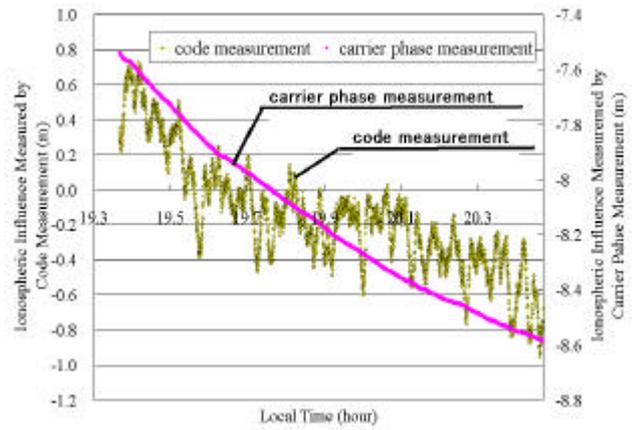


Figure 1. Ionospheric influence measured by raw code and carrier phase measurement

Figure 1 shows that the results of ionospheric influence measured by raw code measurement and carrier phase measurement from dual-frequency GPS receiver. Here that the code measurement is noisy but absolute, and the carrier is very precise but including the cycle ambiguity would be seen.

In the normal, the real-time carrier smoothing is used to measure changes in the ionospheric delay correctly and is estimated the ionosphere delay of time ( $t$ ), it is determined as following:

$$I^t_{smoothing} = w_m^t (I_r)^t + w_n^t (I^t_{smoothing} + (I_f^t - I_f^{t-1})) \quad (12)$$

where  $t$  is the epoch time,  $w_m$  and  $w_n$  are the weighting functions, thus  $w_m + w_n = 1$ . We set

$$w_m^t = w_m^{t-1} + 0.001 \quad \text{and} \quad I^0_{smoothing} = I^0_r.$$

In our research, the post-processing measurements is used to estimate “phase leveling” result of the ionospheric delay of the

past time ( $K$ ), the post-processing measurement is achieved as following:

$$I_{post}^t = I_f^t - \frac{1}{K} \sum_{k=0}^K (I_f^k - I_r^k) \quad (13)$$

where  $t$  is the epoch time,  $I_{post}$  is the result estimated by the post-processed measurement;  $K$  is the continuous time for one satellite in visible, we set  $K=2$ hour in our research.

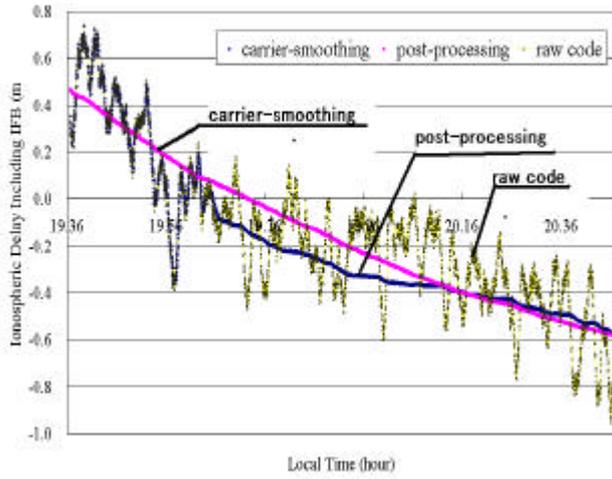


Figure 2. Ionosphere delay measured by three GPS ionosphere measurement

Shown in figure 2 is three GPS ionosphere measurements from a single GPS dual-frequency receiver for one hour: the raw pseudorange difference, carrier-smoothing (step=0.001) and post-processed measurement. From figure 2, it could be seen that the carrier ambiguity could be resolved by post-processed measurement and the “phase leveling” result of the past time could be estimated. However, It is impossible that the ionosphere delay is minus in fact, so the results by measurement were including IFB could be known.

### IFB CALIBRATION

In this section, using the “phase leveling” results measured by Post-processing measurement, SLM assumption and the weighted least squares are used to calibrated IFB and measure vertical TEC.

### SLM Assumption

The ionosphere is a region of ionized plasma that extends from

roughly 50km to 2000km surrounding the surface of the earth. The ionosphere can be usually divided into D, E, F1, F2 and H+ regions according to the electron density, thus the F2 regions (210-1000km) is the most dense and also the highest variability, causing most of the potential effects on GPS receiver system. So we pointed on the TEC of F2 regions to dis cuss the ionosphere effect on GPS system in this paper. For our research, the single layer model (SLM) assumption was proposed to convert the slang TEC (STEC) into the vertical TEC (VTEC) by mapping function  $sl(E)$  [11]. In the single lay model the ionosphere’s total electron content is assumed to consider within an infinitesimal thin shell enveloping the Earth as hollow sphere at a height above the Earth’s mean surface, The relationship between the VTEC and STEC are given as follows:

$$STEC^t = sl(E) \times VTEC^t \quad (14)$$

$$\text{where: } sl(E) = \frac{1}{\cos \left[ \arcsin \left( \frac{R}{R+h} \right) \cos(E) \right]} \quad (15)$$

Here  $E$  is the elevation angle of the line-of sight from the receiver to the satellite,  $R$  is the earth radius,  $h$  is the height of the assumed thin shell of ionosphere above the earth’s surface, In this paper  $h=400$ km because the TEC of F2 region would be discussed .

By Eq.(13) the “phase leveling” quantities of  $I^t$  could be estimated, but we have to note that the results of true measurement are still corrupted by the inter-frequency bias (IFB), given as follows in epoch time  $t$  by Eq.(7) and the IFB is seen as constant here:

$$I_{post}^t = STEC^t + IFB_i \quad (16)$$

where  $t$  is epoch time,  $i$  is the PRN of the GPS satellite,

$I_{post}$  is the results by post-processed measurement. It is noted that here the multi-path effect is negligible because the differential carrier phase is much less sensitive to multi-path [13].

From Eq.(14) and Eq.(16), the equation can be written as following:

$$I_{post}^t = sl(E)_i \times VTEC^t + IFB_i \quad (17)$$

Here It has to be noted that the VTEC is not simply as the SLM assumption in fact, and the error by mapping function  $sl(E)$  will

be increased at the low elevation angle [1]. So the highest and the second highest elevation satellite in the continuous observation time will be chosen in our measurement. One more reason to choose the high elevation satellite is to reduce the multi-path influence [13].

### IFB Estimation

To reduce the errors caused by SLM assumption and influence of multi-path, we chose two satellites of the highest and the second highest elevation in the average for 2 hours. By two satellites, we calculated the average of the one-minute average of  $I_{post}$  measured by post-processed measurement and  $sl(E)$

measured by SLM assumption. The following equation will be derived by Eq.(13)

$$\overline{(1/sl(E))_i} \times \overline{I_{post_i}} = \overline{VTEC} + \overline{(1/sl(E))_i} \times IFB_i \quad (17)$$

where  $i$  is PRN of satellite,  $\overline{(1/sl(E))_i}$  is minutely average of  $sl(E)$ ,  $\overline{I_{post_i}}$  is minutely average of results estimated by post-processed measurement.

Because VTEC measured by each satellites are the same in fact, the unknown terms  $IFB_i$  can be determined by two chosen satellites through the weighted least squares with minimize the residual error as followings:

$$\Delta I_{post_{12}} = \left[ \overline{(1/sl(E))_1} \times \overline{I_{post_1}} \right] - \left[ \overline{(1/sl(E))_2} \times \overline{I_{post_2}} \right] \quad (18)$$

$$R_{err} = \sum_1^T W^t \left[ \Delta I_{post_{12}} - \left( \overline{(1/sl(E))_1} \times IFB - \overline{(1/sl(E))_2} \times IFB \right) \right]^2 \quad (19)$$

where 1, 2 indicate the highest and the second highest elevation satellites;  $T=120$  in this paper.  $W^t$  is the weighting function to reduce the estimating error. In this method,

$W^t = \left( \overline{(1/sl(E))_1} - \overline{(1/sl(E))_2} \right)^2$ , it is set to reduce the error by mapping function  $sl(E)$ . With IFB was measured, one minute average VTEC could be estimated simply by Eq.(17).

### RESULTS OF EXPERIMENT

To test the result of this method, the experiment was done to measure the VTEC of 400km height. One antenna was set on the roof of the building in the Tokyo University of Mercantile

Marine ( $N 35^\circ 39'$ ,  $S 139^\circ 47'$ ), by the GPS accessories, the signal was separated and supplied to two receivers (Novetal RT2, receiver1, receiver2), the GPS data was logged from 21:00 April 7, 2003 to 07:00 April 8, 2003 (Local Time) for 10 hours with sampling one second. By the proposed method, the VTEC could be measured with receiver1 and receiver2. Because signal was received by one antenna, the VTEC measured by two receivers should be same in fact. We compared the difference between VTEC of receiver1 and receiver2, and found that the difference is less than 0.03m. Finally the results of the experiment was compared with the data from IRI-95 model, the difference is less than 0.2m.

### IFB Influence

Because the same antenna received GPS data, the multi-path errors and the ionospheric delay measured by two receivers should be same in this experiment.

Figure 3 shows measured by post-processed measurement from two GPS receivers through one antenna using PRN 23 satellite for 2 hours. Two GPS receivers by one antenna would measure the exact same ionosphere to satellite PRN23 in the same time. So the relative IFB here is obvious.

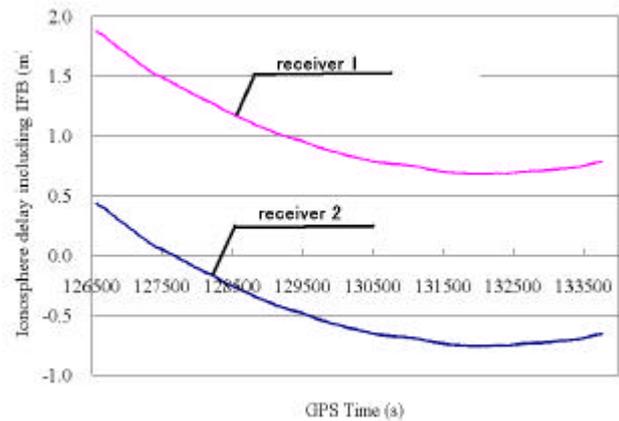


Figure 3. Ionosphere delay measured by receiver1 and receiver2

### IFB Estimation

Now the proposed method was used to determine IFB from two receivers separately, and one-minute average VTEC for 10 hours would be estimated.

In the first the highest and second highest satellite in the average for 2 hours were chosen. Table 1 shows PRN and two hours

average of elevation of two satellites, which are chosen into our research. Here the elevation of chosen satellites are more than 45 degree would be seen.

Table 1. PRN and 2 hour average elevation of GPS satellites chosen in the experiment

L.T (hour)	PRN satellite	elevation (degree)	PRN satellite	elevation (degree)
2100-2300	PRN 23	62.0	PRN 18	58.8
2300-0100	PRN 30	67.3	PRN 5	59.7
0100-0300	PRN 14	67.5	PRN 25	48.8
0300-0500	PRN 25	70.9	PRN 16	52.9
0500-0700	PRN 3	66.4	PRN 16	59.4

It must be noted that the multi-path influence in the dual-frequency measurement, so the higher elevation satellites have to be chosen. Because the elevations of chosen satellite are more than 45 degree, and also, the post-processing measurement was used in the first step, the error by multi-path would be negligible. Using the chosen satellites, the IFB of these satellites could be calibrated by the weighted least squares in the experiment. Table 2 shows that IFB of chosen satellites measured by the weighted least squares from two GPS receivers.

Table 2. The IFB of chosen satellites measured by two receivers

GPS satellite	IFB measured by receiver 1 (m)	IFB measured by receiver 2 (m)
PRN 23	0.246	2.686
PRN 18	1.840	4.402
PRN 5	1.251	3.668
PRN 30	0.593	3.077
PRN 14	2.651	5.198
PRN 25	1.993	4.446
PRN 16	4.387	6.946
PRN 3	2.196	5.089

By calibrating IFB of chosen satellites, the minutely VTEC of two receivers for 10 hours could be measured simply.

### Compare Vertical TEC measured by two receivers

In this experiment, the quantities of vertical TEC measured from two GPS receivers are the same, so by comparing results of two receivers, the valid of the new method could be seen.

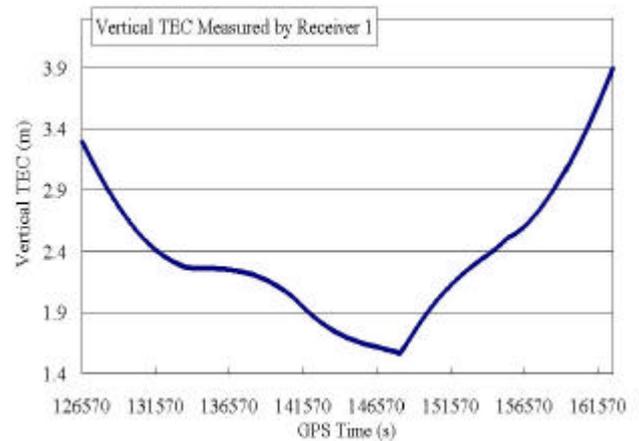


Figure 4. Vertical TEC measured by receiver 1

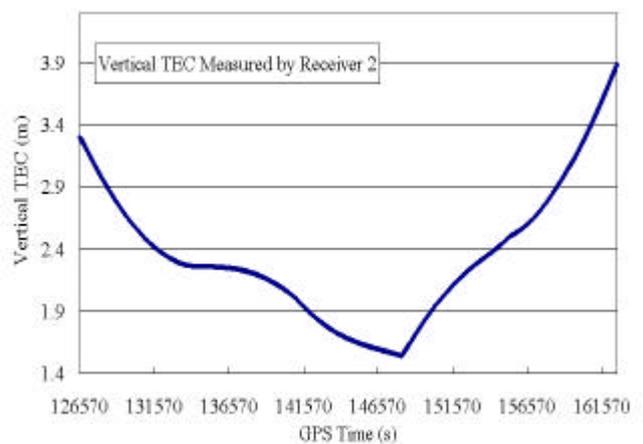


Figure 5: Vertical TEC measure by receiver 1

The figure 4 and figure 5 show the minutely average vertical TEC measured by TEC measurement from receiver1 and receiver 2 from 21:00 April 7 2003 to 07:00 April 8, 2003 (Local Time) for 10 hours.

By the difference between the VTEC measured from two receivers calculated, the valid of the method would be seen. Figure 6 shows that the difference between VTEC measured by receiver 1 and receiver 2, we could see that the biggest of the difference is only about 0.027m here.

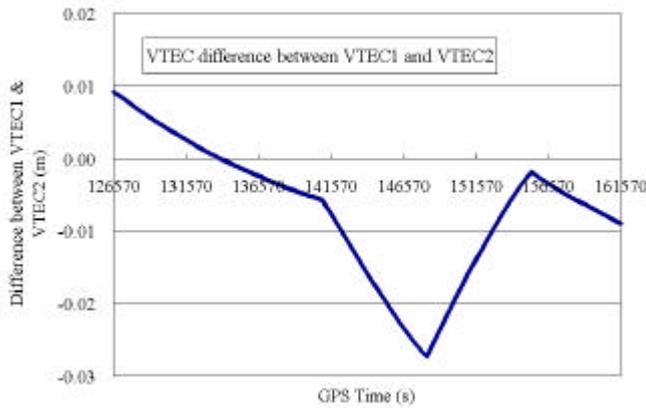


Figure 6. VTEC difference between VTEC1 and VTEC2

### Compare With IRI-95 Model

To test this proposed method, the VTEC measured by receiver1 is compared with the International Reference Ionosphere (IRI-95) model. The IRI was developed by an international project sponsored by the Committee on space Research (COSPAR) and the International Union of Radio Science (URSI) [9]. For a given location, time, and data, the IRI describes many ionospheric variables, including the electron density, for a valid range of altitudes below 1000 km. Tests proved that IRI-95 model performs better than IRI-90 model in computing the ionospheric delay for the single-frequency altimeters [10]. Here we set the height is 400m in IRI-95 model.

Table 3: results from TEC measurement and IRI-95 model

Local Time (hour)	IRI-95 (m)	VTEC1 (m)	Difference (m)
21.0(7 <sup>th</sup> )	3.41	3.30	+0.11
22.0	2.72	2.59	+0.13
23.0	2.30	2.26	+0.04
24.0	2.24	2.23	+0.01
1.0(8 <sup>th</sup> )	1.90	1.88	+0.02
2.0	1.70	1.68	+0.02
3.0	1.51	1.56	-0.05
4.0	1.95	2.10	-0.15
5.0	2.16	2.39	-0.23
6.0	2.99	2.97	+0.02
7.0	4.04	3.88	+0.16

Table 3 shows that IRI-95 and results of receiver 1 measured by TEC measurement from 21:00 April 7 to 07:00 April 8 2003. It would be seen that there is about 0.094m (0.6TECU) differences in average between IRI-95 and VTEC measured by

receiver1 from table 3. That the TEC measurement can measure the vertical TEC validly would be known. Here we think that converting the slant TEC into the vertical TEC with SLM assumption causes the difference, because actually electron density distribution is not thin shell as SLM assumption and includes higher (plasmasphere) and lower (E region). Calibrating the error of SLM assumption is our research in the future.

### CONCLUSION

We have made the new two-step method to estimate one minute VTEC by single GPS dual-frequency receiver. In the first step, using the first-order refraction index, we made the post-processing measurement to calibrate the relative ambiguity, the changes of ionospheric delay could be measured correctly and the "phase leveling" results could be estimated. But the results include IFB. In the second step, Using SLM assumption, STEC could be converted into VTEC. Because IFB are constant over several days, a weighted least square was used to determine the IFB, and VTEC could be measured.

The new method was tested by the experiment from 2100LT 7<sup>th</sup> to 0700LT 8<sup>th</sup> April. Two GPS receivers received GPS signal by one antenna, VTEC were measured by the proposed method with receiver1 and receiver2. By calculating the difference between measured VTEC, that the difference is very little could be seen. In the finally we compare VTEC of receiver 1 with the results of IRI-95 model, and about 0.09m difference in average could be known. This method could measure the vertical TEC by single GPS receiver.

### REFERENCE

- [1] Klobuchar. J.A., Global Positioning System: Theory and Applications, American Institute of American Institute of Aeronautics and Astronautics, Inc. 1996
- [2] Coco D., GPS-Satellite of opportunity for ionospheric monitoring. GPS World, October, 1991
- [3] Bishop, G. J., Coco D.S., N.Lunt, Coker C., Mazzella A.J., and Kersley L., Application of SCORE to extract protonospheric electron content from GPS/NNSS observations, in Proceedings of ION GPS 97, pp. 207-216, Inst. Of Navig., Alexandria, Va., 1997.
- [4] Bilitza, D., Rawer, K., Bossy, L and Gulyaeva, T., International Reference Ionosphere-Past, Present and Future: I Electron Density. Journal of Adv. Space Research, Vol.13, No. 3, pp(3)3-(3)13, 1993.

- [5] Abbott, M. M., and Van Ness, H. C., Thermodynamics, McGraw Hill, New York, 1989.
- [6] Davies, K., Ionospheric Radio, Peter Peregrinus, Ltd., London, 1989.
- [7] Brunner, F, K., and Gu, M., "An Improved Model for the Dual Frequency Ionospheric Correction of GPS Observations," Manuscripta Geodaetica, Vol. 16, pp. 280-289, 1991.
- [8] Ya. L. Al'pert., Radio Wave Propagation And The Ionosphere, A Division of Plenum Publishing Corporation, New York, 1973
- [9] Otsuka Y., Ogawa T., Saito A., Tsugawa T., Fukao S., and Miyazaki S., A new technique for mapping of total electron content using GPS network in Japan, Earth Planets Space, 54, pp. 63-70, Japan, 2001.
- [10] Sardon, E., A. Rius, and N.Zarroa, Estimation of the transmitter and receiver differential biases and the ionospheric total electron content from Global Positioning System observations, Radio Sci., 29, pp.577-586, 1994.
- [11] Gao, Y., P. Heroux and J. Kouba, Estimation of GPS Receiver and Satellite L1/L2 Signal Delay Bias Using from CACS, Proceedings of KIS-94, Banff, Canada, August 30-September 2, 1994.
- [12] Gao, Y., and Z. Z. Liu, Precise Ionosphere Modeling using Regional GPS Network Data, Journal of Global Positioning System Vol.1, No.1:18-24, 2002
- [13] Hatch. R., "The synergism of GPS Code and carrier Measurement," Magnavox Technical Paper, MX-TM-3353-82, Jan. 1982.