



国立大学法人

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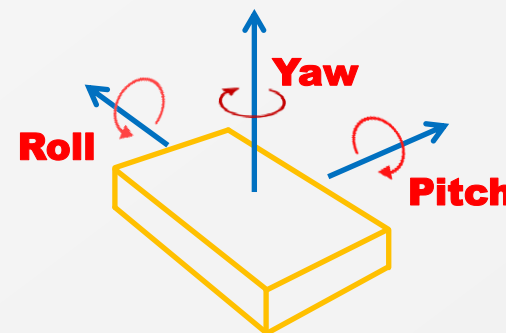
Tokyo University of Marine Science and Technology



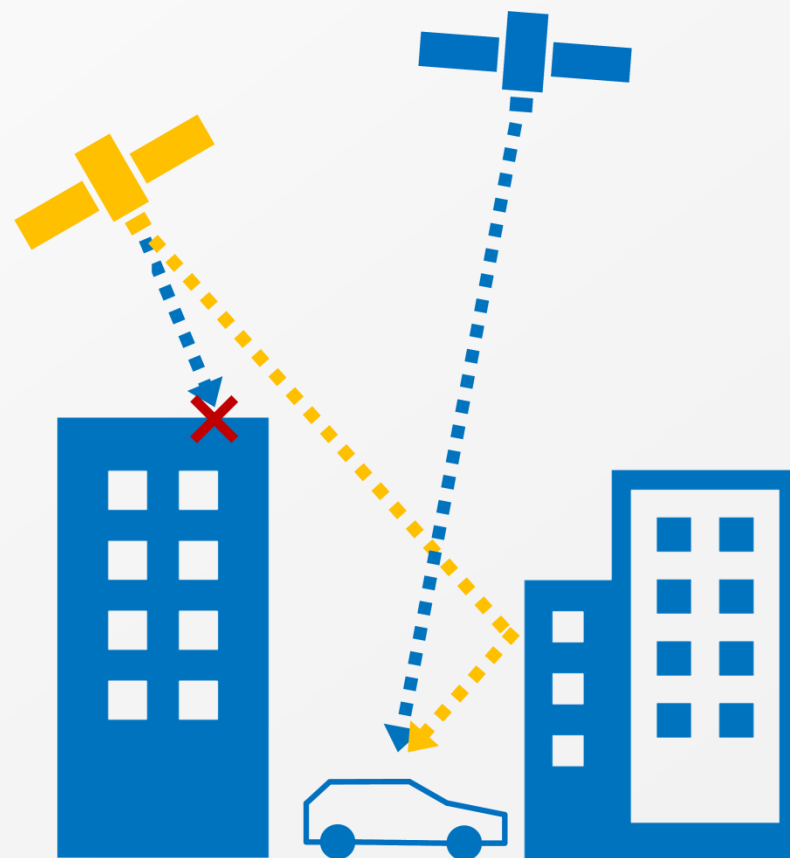
都市部における GNSS 単独測位性能改善に関する研究

2019/02/15 情報通信工学研究室 富永貴樹

- High demand for precise and robust GNSS navigation in the automotive market.
  - applications such as Autonomous Driving, ADAS, and V2X.
- GNSS SPP (Single Point Positioning) is still important for the “robust” navigation.
  - PPP/RTK lost compensation
  - Sensor calibration



- Implemented adaptive EKF to improve the GNSS SPP performance of a mass product receiver in an urban environment.
  - Performance : accuracy and precision, and its integrity.
  - This is the challenge in the urban canyon because of NLOS (Non-Line-Of-Sight) signal tracking.

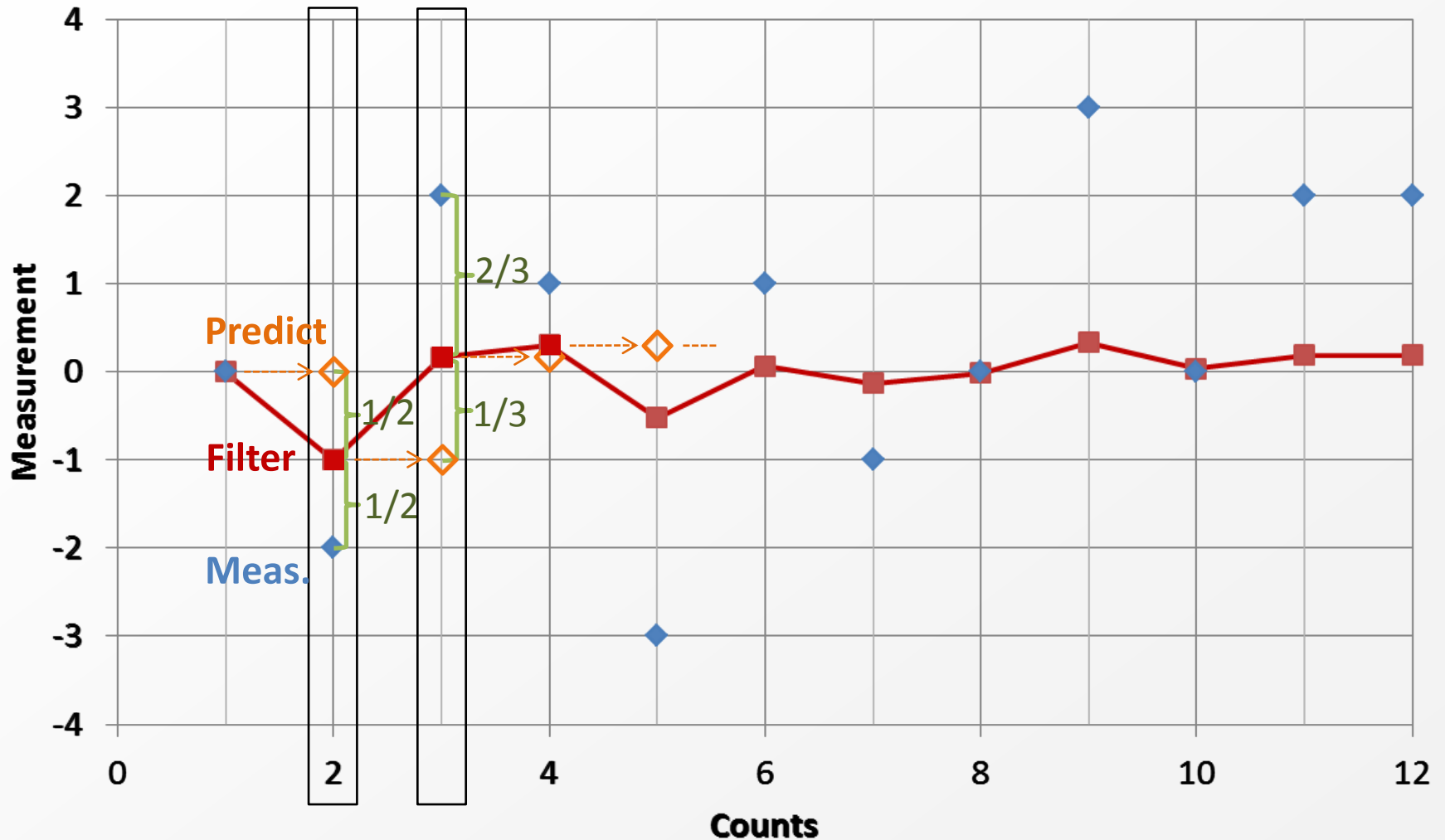


- **Introduction to Adaptive Kalman Filter**
  - Kalman filter
  - IAE(Innovation-based Adaptive Estimation)
  - Applying IAE to GNSS
- **Adaptive EKF vs. Urban Canyon**
  - Positioning accuracy and precision
  - Measurement error and predicted sigma
  - Integrity Information
- **Conclusion and future works**

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# Hatch filter

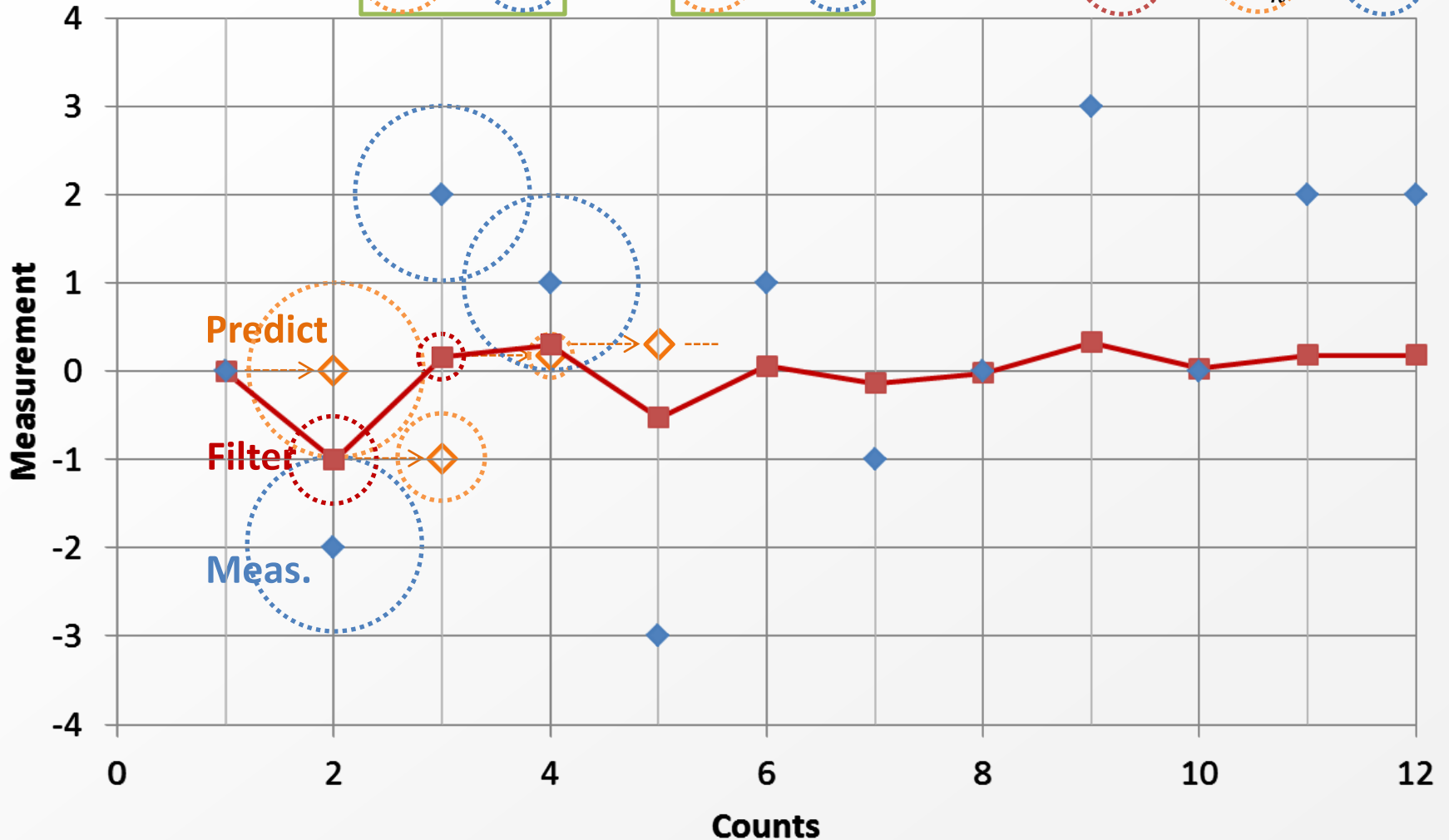
$$x_k = \frac{1}{M} z_k + \frac{M-1}{M} x_{\bar{k}}$$



# Weighted-LS-based filter

$$x_k = \frac{\sigma_{x_{\bar{k}}}^2}{\sigma_{x_{\bar{k}}}^2 + \sigma_{z_k}^2} z_k + \frac{\sigma_{z_k}^2}{\sigma_{x_{\bar{k}}}^2 + \sigma_{z_k}^2} x_{\bar{k}}$$

$$\frac{1}{\sigma_{x_k}^2} = \frac{1}{\sigma_{x_{\bar{k}}}^2} + \frac{1}{\sigma_{z_k}^2}$$



- **Weighted-LS:**

$$x_k = \frac{\sigma_{x_{\bar{k}}}^2}{\sigma_{x_{\bar{k}}}^2 + \sigma_{z_k}^2} z_k + \frac{\sigma_{z_k}^2}{\sigma_{x_{\bar{k}}}^2 + \sigma_{z_k}^2} x_{\bar{k}}$$

$$\sigma_{x_k}^2 = \frac{\sigma_{z_k}^2 \sigma_{x_{\bar{k}}}^2}{\sigma_{x_{\bar{k}}}^2 + \sigma_{z_k}^2}$$

- **Defining:**

$$K_k = \frac{\sigma_{x_{\bar{k}}}^2}{\sigma_{z_k}^2 + \sigma_{x_{\bar{k}}}^2}$$

- **Then,**

$$x_k = x_{\bar{k}} + K_k (z_k - x_{\bar{k}})$$

$$\sigma_{x_k}^2 = (1 - K_k) \sigma_{x_{\bar{k}}}^2$$

Measurement Update

- **And,**

$$\sigma_{x_{\bar{k}}}^2 = \sigma_{x_{k-1}}^2 + \sigma_{\varepsilon}^2$$

$$x_{\bar{k}} = x_{k-1} + v_k$$

Time Update



- Summary for Kalman filter

**Prediction**  
**(Time update)**

$$\sigma_{x_{\bar{k}}}^2 = \sigma_{x_{k-1}}^2 + \sigma_{\varepsilon}^2$$
$$x_{\bar{k}} = x_{k-1} + v_k$$

$$K_k = \frac{\sigma_{x_{\bar{k}}}^2}{\sigma_{x_{\bar{k}}}^2 + \sigma_{z_k}^2}$$
$$x_k = x_{\bar{k}} + K_k(z_k - x_{\bar{k}})$$
$$\sigma_{x_k}^2 = (1 - K_k)\sigma_{x_{\bar{k}}}^2$$

**Filter**  
**(Measurement Update)**

**Measurement**

$$\sigma_{z_k}^2$$
$$z_k$$

epoch  $k$

- **Non-linear system**

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \boldsymbol{\varepsilon}$$

– By Taylor series,

$$\mathbf{h}(\mathbf{x}) = \mathbf{h}(\mathbf{x}_0) + \mathbf{H}(\mathbf{x} - \mathbf{x}_0) + \dots$$

$$\mathbf{H} = \left. \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_0}$$

– Linearize :  $\mathbf{z} \sim \mathbf{h}(\mathbf{x}_0) + \mathbf{H}(\mathbf{x} - \mathbf{x}_0) + \boldsymbol{\varepsilon}$

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{H}^{-1}(\mathbf{z} - \mathbf{h}(\mathbf{x}_0)) + \boldsymbol{\varepsilon}$$

- **EKF formulation,**

$$\mathbf{x}_k = \mathbf{x}_{\bar{k}} + \mathbf{K}_k(\mathbf{z} - \mathbf{h}(\mathbf{x}_{\bar{k}}))$$

$$\left( \lim_{\mathbf{R}_k \rightarrow 0} \mathbf{K}_k = \mathbf{H}^{-1} \right)$$

- Summary

**Predictition**  
**(Time update)**

$$\begin{aligned}\mathbf{x}_{\bar{k}} &= \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{D}_k \mathbf{u}_k \\ \mathbf{P}_{\bar{k}} &= \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_{k-1}^T + \mathbf{Q}_k\end{aligned}$$

$$\begin{aligned}\mathbf{K}_k &= \frac{\mathbf{P}_{\bar{k}} \mathbf{H}(\mathbf{x}_{\bar{k}})^T}{(\mathbf{H}(\mathbf{x}_{\bar{k}}) \mathbf{P}_{\bar{k}} \mathbf{H}(\mathbf{x}_{\bar{k}})^T + \mathbf{R}_k)} \\ \mathbf{x}_k &= \mathbf{x}_{\bar{k}} + \mathbf{K}_k (\mathbf{z}_k - \mathbf{h}(\mathbf{x}_{\bar{k}})) \\ \mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}(\mathbf{x}_{\bar{k}})) \mathbf{P}_{\bar{k}}\end{aligned}$$

**Filter**  
**(Measurement Update)**

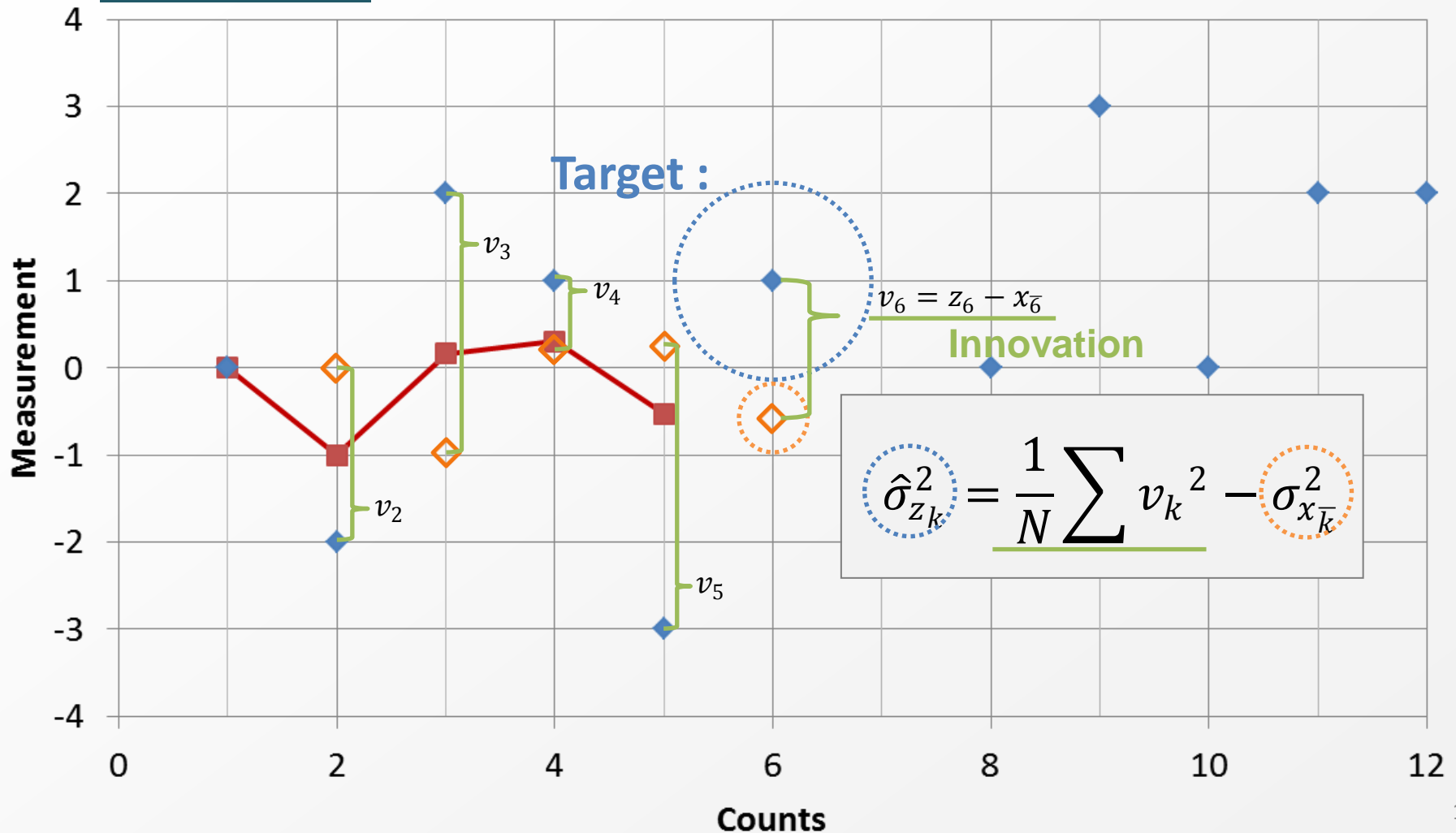
**Measurement**

$$\begin{aligned}\mathbf{z}_k \\ \mathbf{R}_k\end{aligned}$$

epoch  $k$

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- Run KF with appropriate  $\sigma_{z_k}^2$ , which is unknown.



- Summary for IAE

**Prediction**  
**(Time update)**

$$\sigma_{x_{\bar{k}}}^2 = \sigma_{x_{k-1}}^2 + \sigma_{\varepsilon}^2$$
$$x_{\bar{k}} = x_{k-1} + v_k$$

$$K_k = \frac{\sigma_{x_{\bar{k}}}^2}{\sigma_{x_{\bar{k}}}^2 + \hat{\sigma}_{z_k}^2}$$
$$x_k = x_{\bar{k}} + K_k (z_k - x_{\bar{k}})$$
$$\sigma_{x_k}^2 = (1 - K_k) \sigma_{x_{\bar{k}}}^2$$

**Filter**  
**(Measurement Update)**

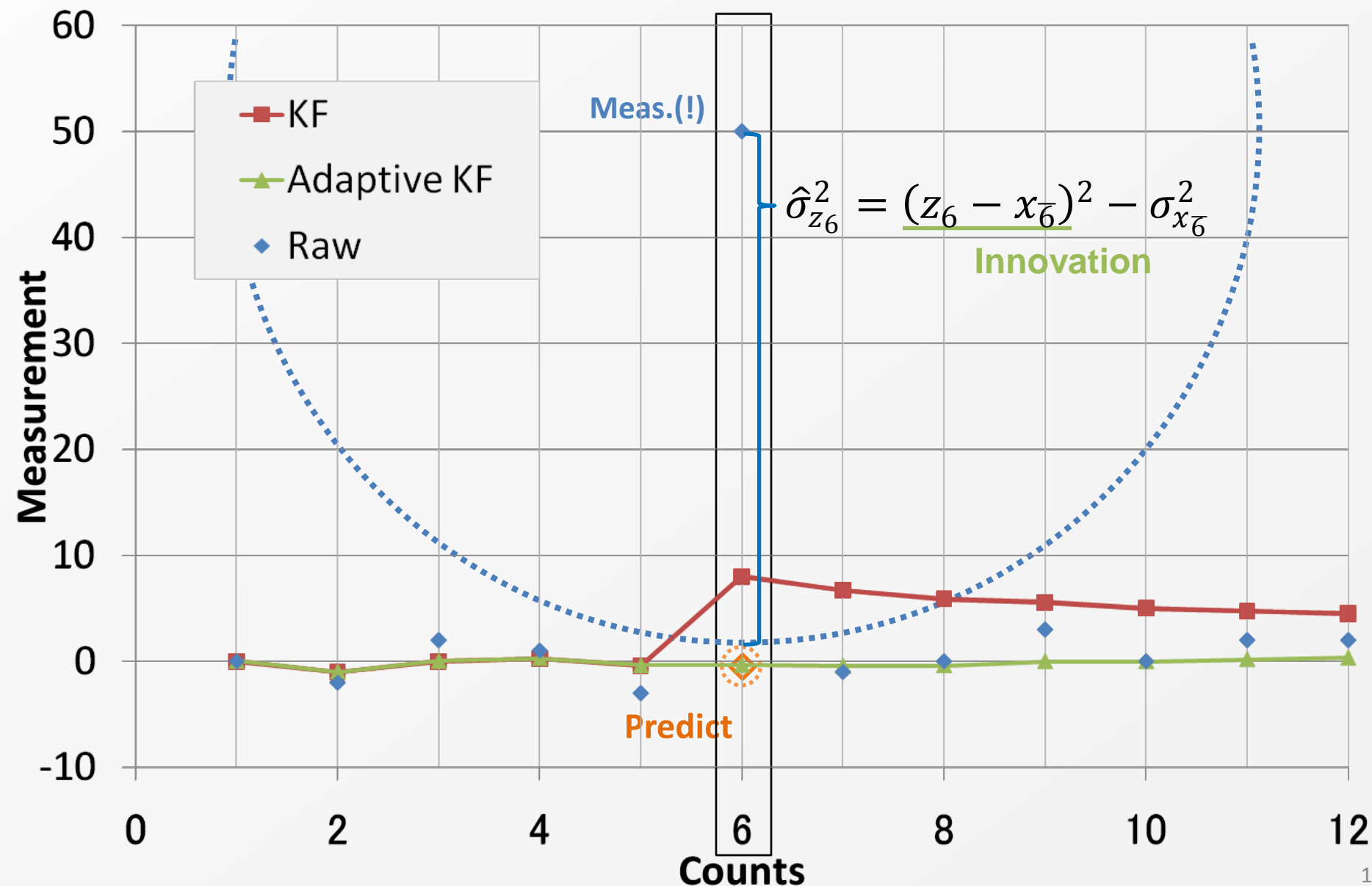
**Measurement**

$$\hat{\sigma}_{z_k}^2$$
$$z_k$$

epoch k

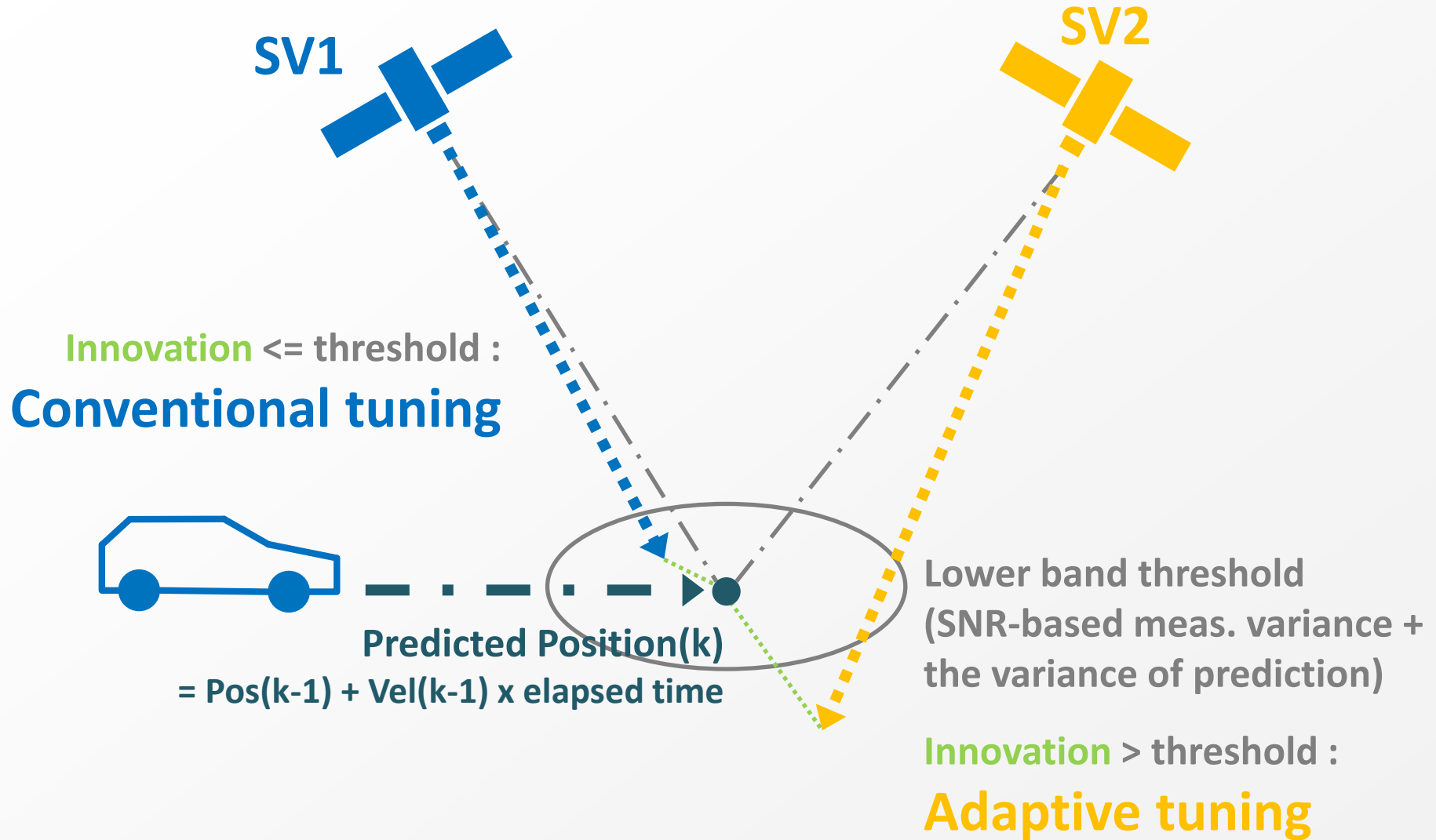
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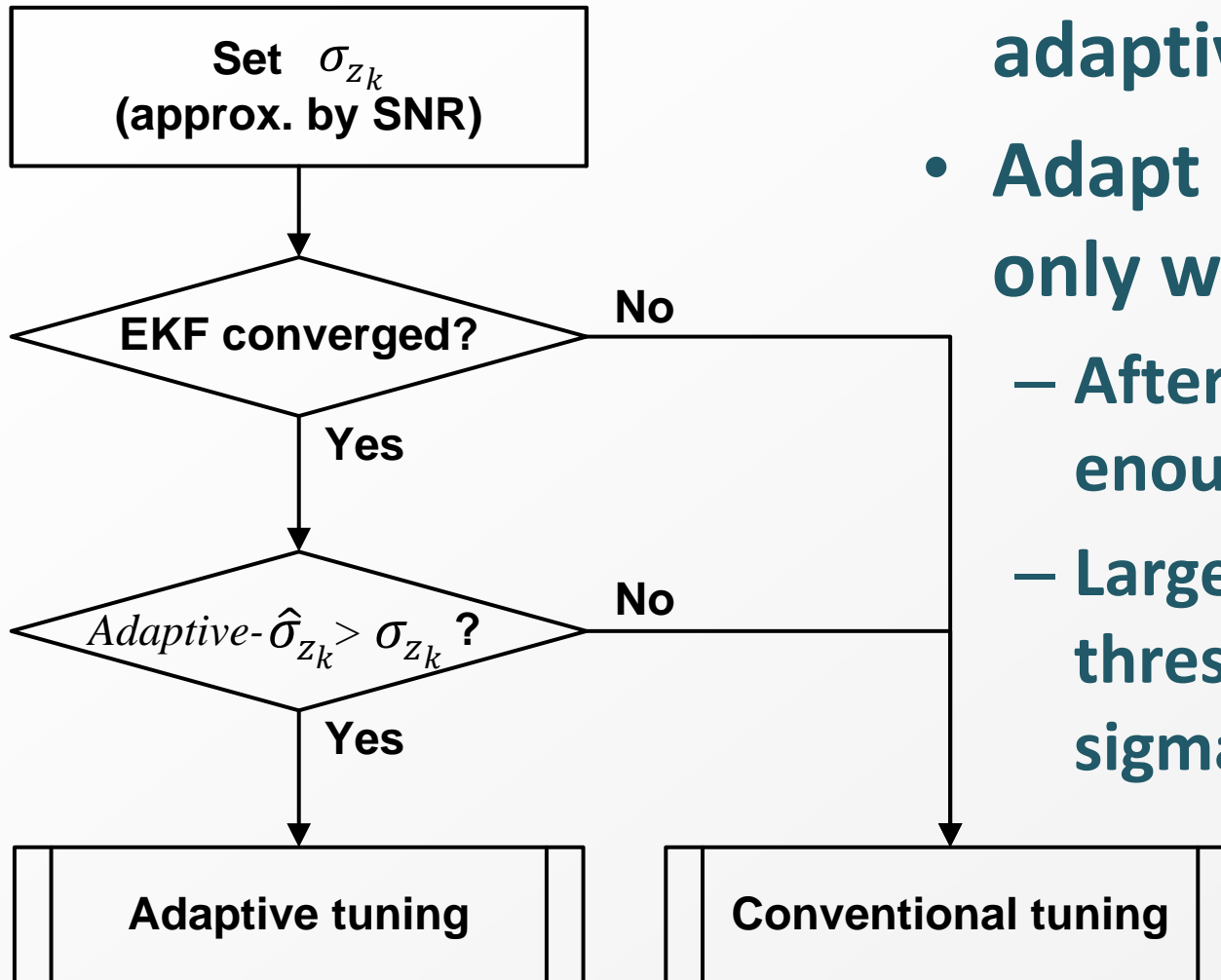
# Applying IAE to Outlier





# Applying IAE to GNSS





- Process flow to create adaptive  $R_k$ .
- Adapt the innovation only when:
  - After EKF converged enough.
  - Larger than lower band threshold (SNR-based sigma).

- Summary for extended-Kalman filter (EKF)

Prediction  
(Time update)

$$\begin{aligned}\mathbf{x}_{\bar{k}} &= \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{D}_k \mathbf{u}_k \\ \mathbf{P}_{\bar{k}} &= \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_{k-1}^T + \mathbf{Q}_k\end{aligned}$$

$$\begin{aligned}\mathbf{K}_k &= \frac{\mathbf{P}_{\bar{k}} \mathbf{H}(\mathbf{x}_{\bar{k}})^T}{\left(\mathbf{H}(\mathbf{x}_{\bar{k}}) \mathbf{P}_{\bar{k}} \mathbf{H}(\mathbf{x}_{\bar{k}})^T + \widehat{\mathbf{R}}_k\right)} \\ \mathbf{x}_k &= \mathbf{x}_{\bar{k}} + \mathbf{K}_k (\mathbf{z}_k - \mathbf{h}(\mathbf{x}_{\bar{k}})) \\ \mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}(\mathbf{x}_{\bar{k}})) \mathbf{P}_{\bar{k}}\end{aligned}$$

Filter  
(Measurement Update)

Measurement

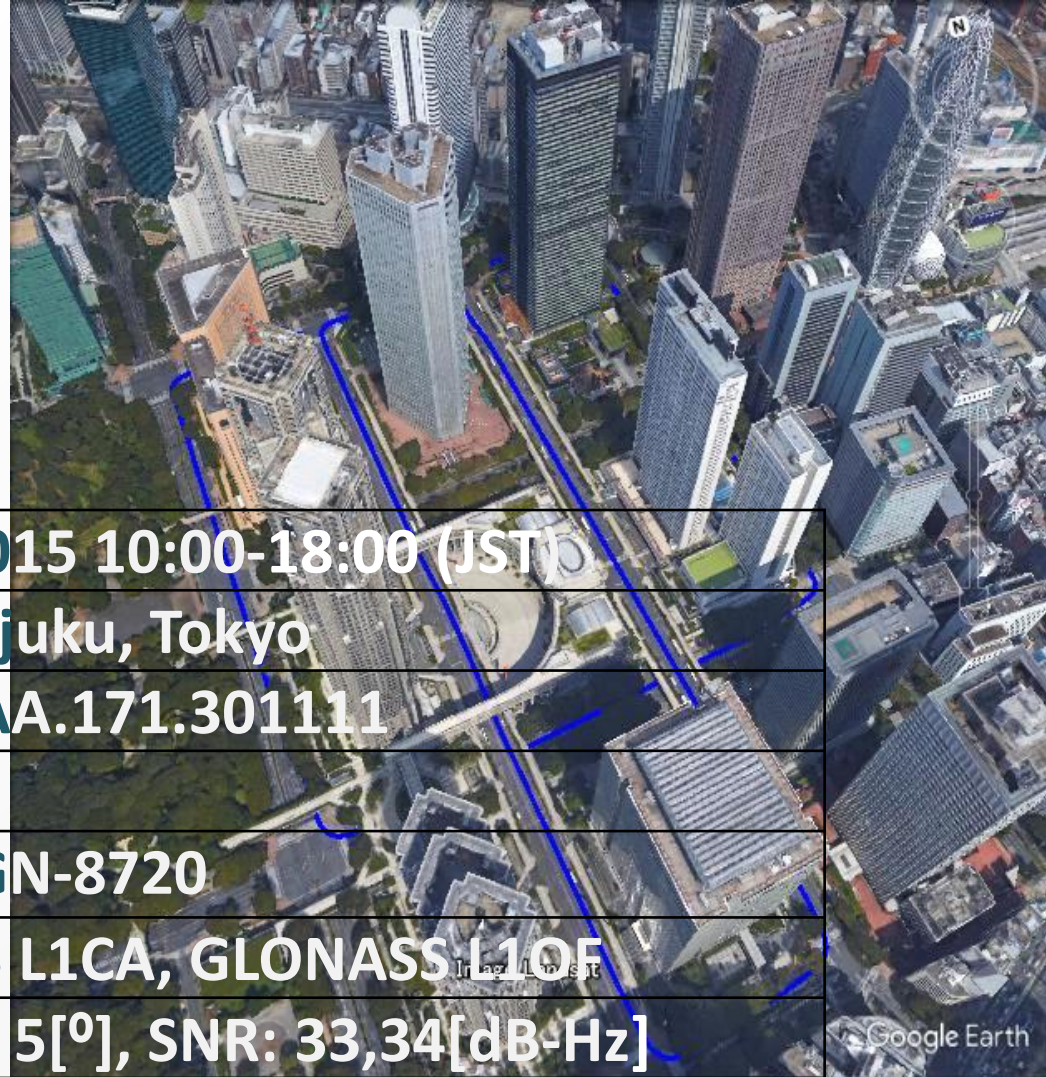
$$\begin{aligned}\mathbf{z}_k \\ \widehat{\mathbf{R}}_k\end{aligned}$$

Adaptive covariance matrix  
of measurement noise

epoch  $k$

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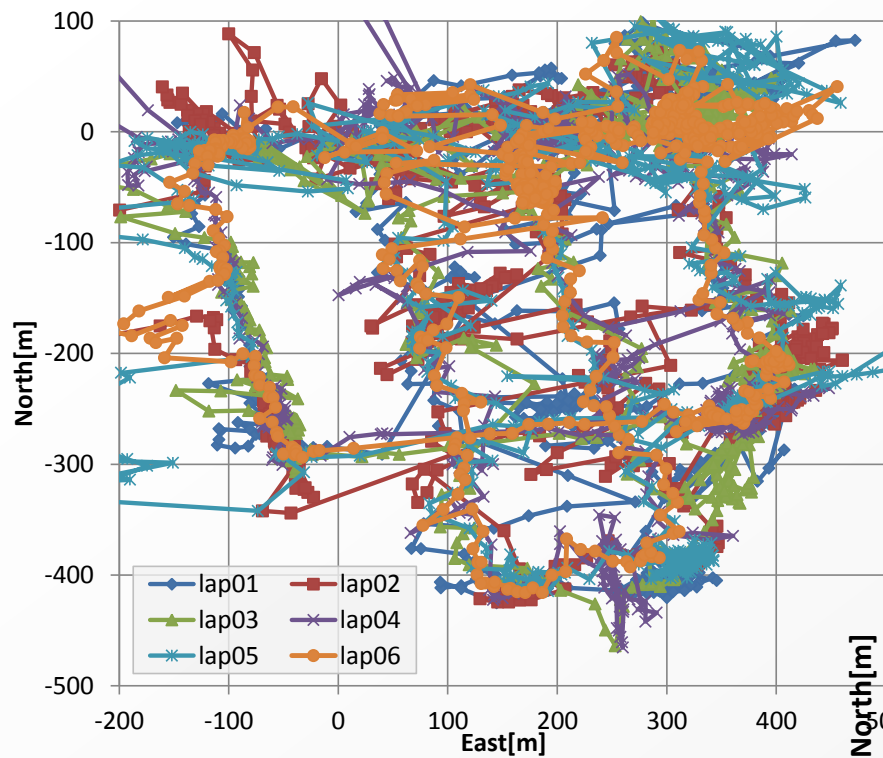
# Urban Challenge



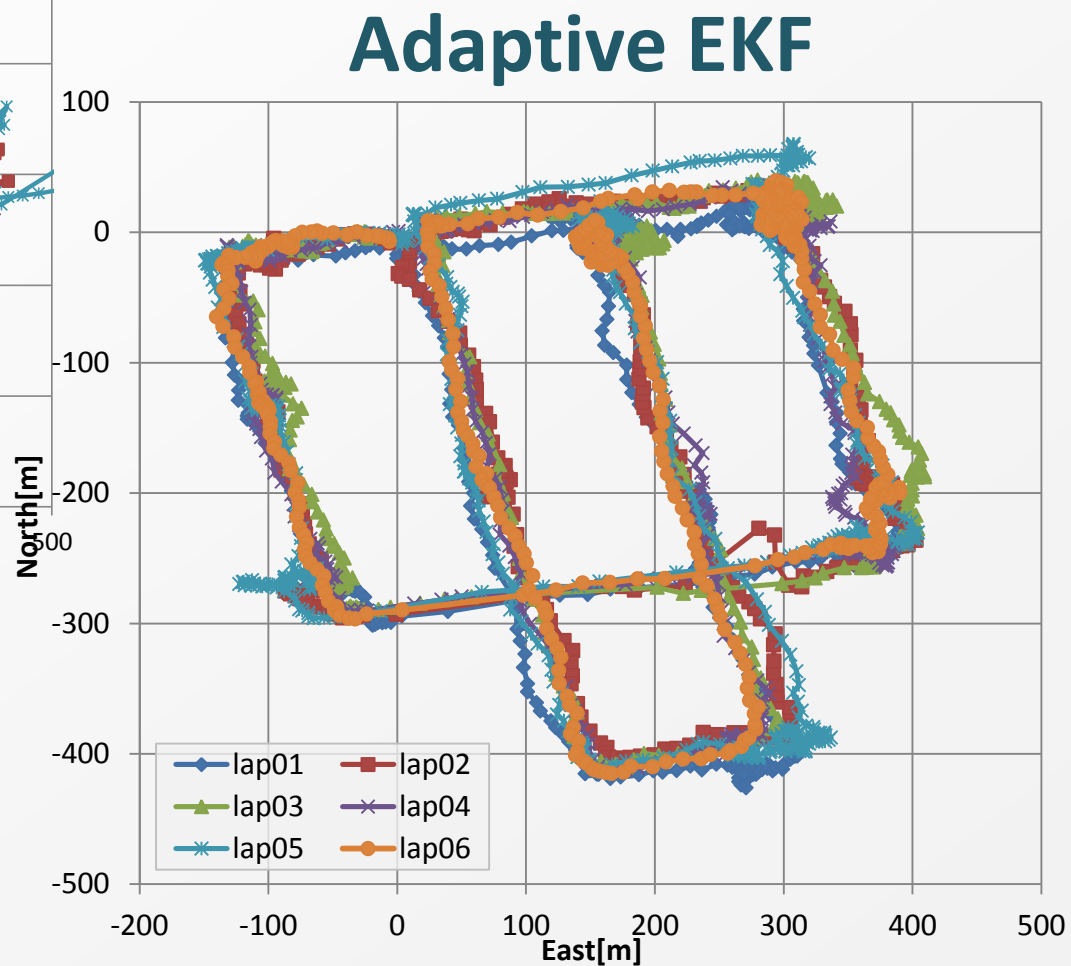
- Test configurations

Date & Time	Nov.9 <sup>th</sup> .2015 10:00-18:00 (JST)
Test Area	Nishi-shinjuku, Tokyo
Antenna Type	Taoglas AA.171.301111
Antenna Place	Car roof
Raw Meas.	Furuno GN-8720
GNSS System	GPS/QZSS L1CA, GLONASS L1OF
Masks	Elevation: 5[°], SNR: 33,34[dB-Hz]
Sampling Rate	1Hz
True Position	Applanix POSLV 520(Post Proc.)
EKF Types	Conventional EKF & Adaptive EKF

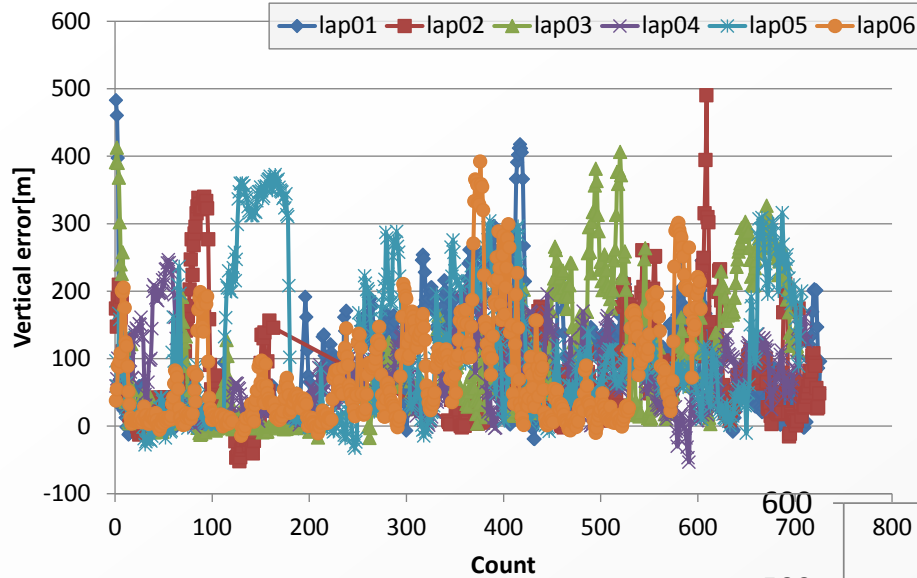
# Horizontal position error



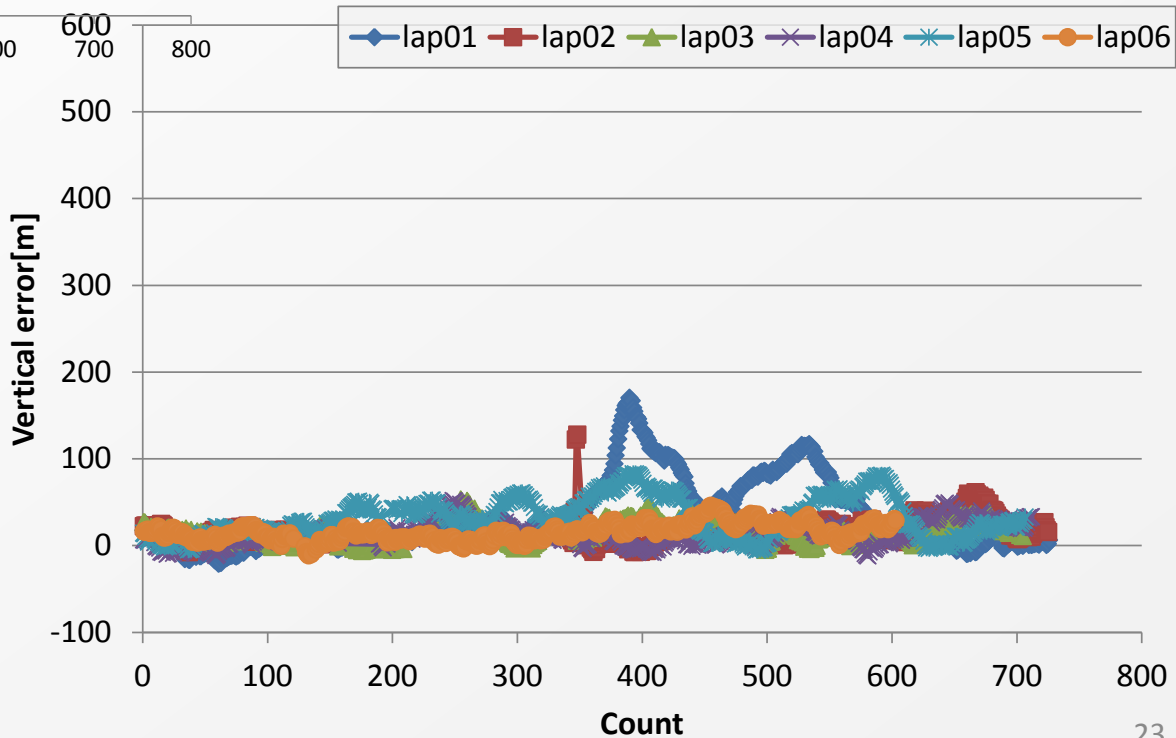
## Conventional EKF



# Vertical position error



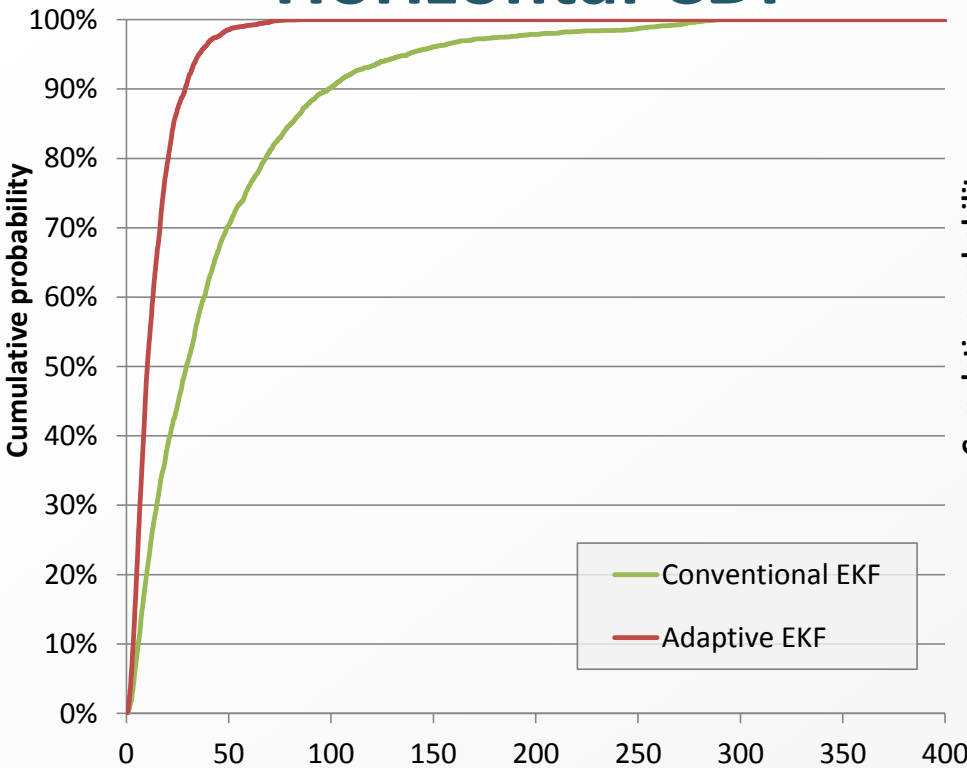
## Conventional EKF



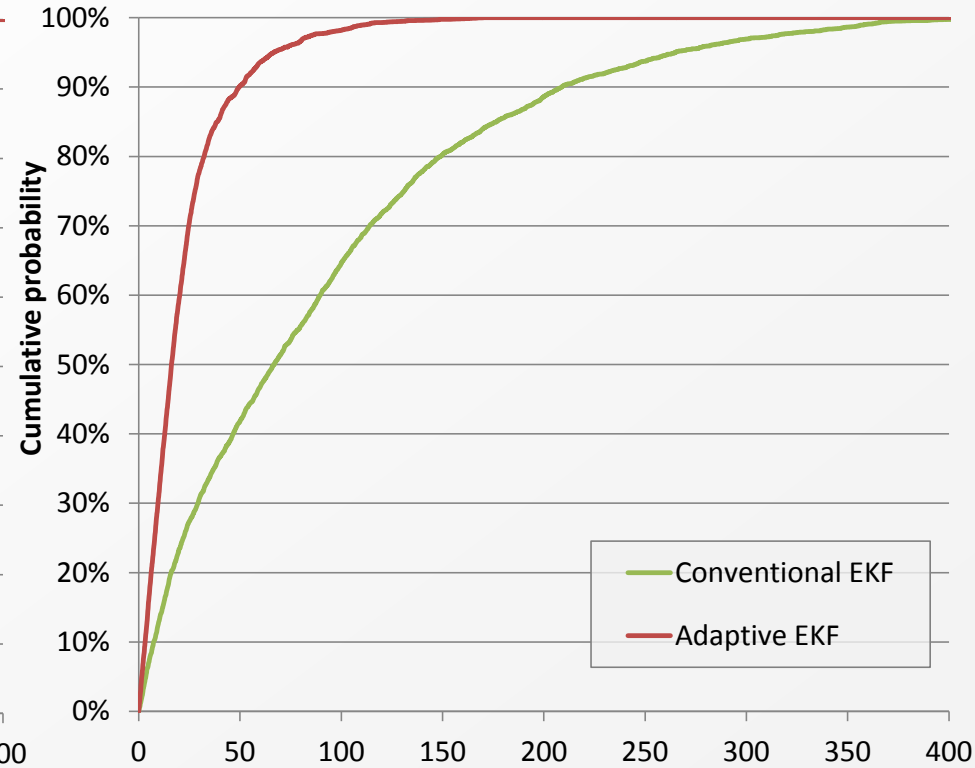
## Adaptive EKF

# Statistics for position error

## Horizontal CDF



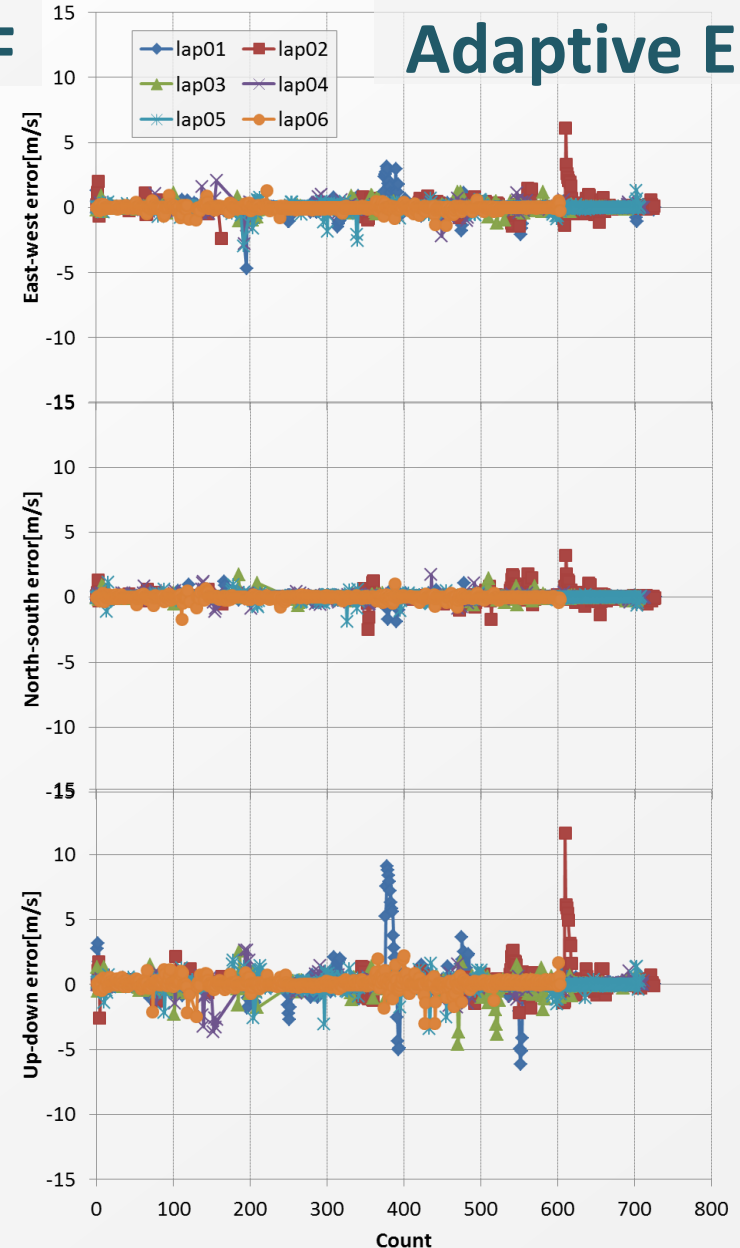
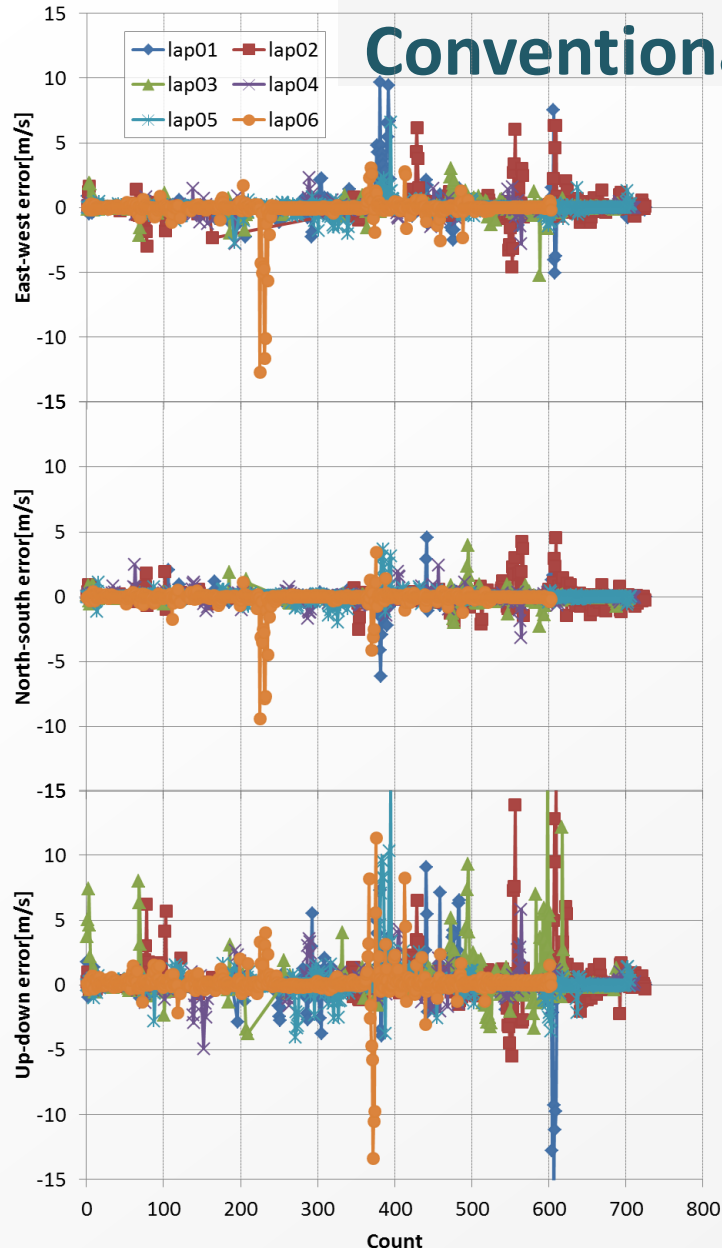
## Vertical CDF



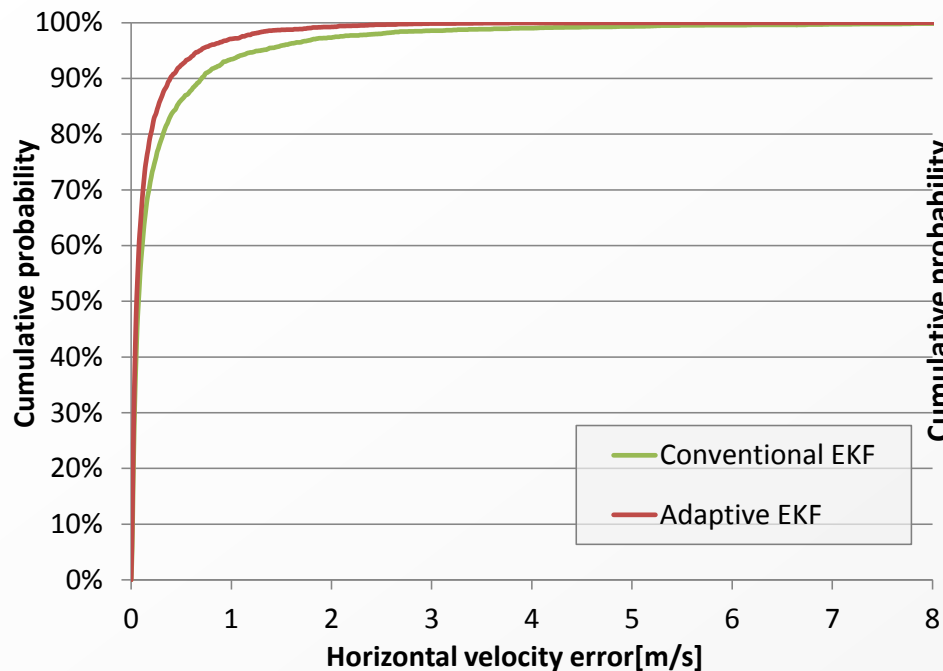
		Mean error[m]	Error at 68.27% [m]	Error at 95.45% [m]	Error at 100% [m]
Horizontal	Adaptive EKF	2.50	15.9	36.6	85.0
	Conventional EKF	13.35	46.7	141.6	290.0
Vertical	Adaptive EKF	21.44	23.9	70.6	170.0
	Conventional EKF	88.89	109.6	272.9	490.2



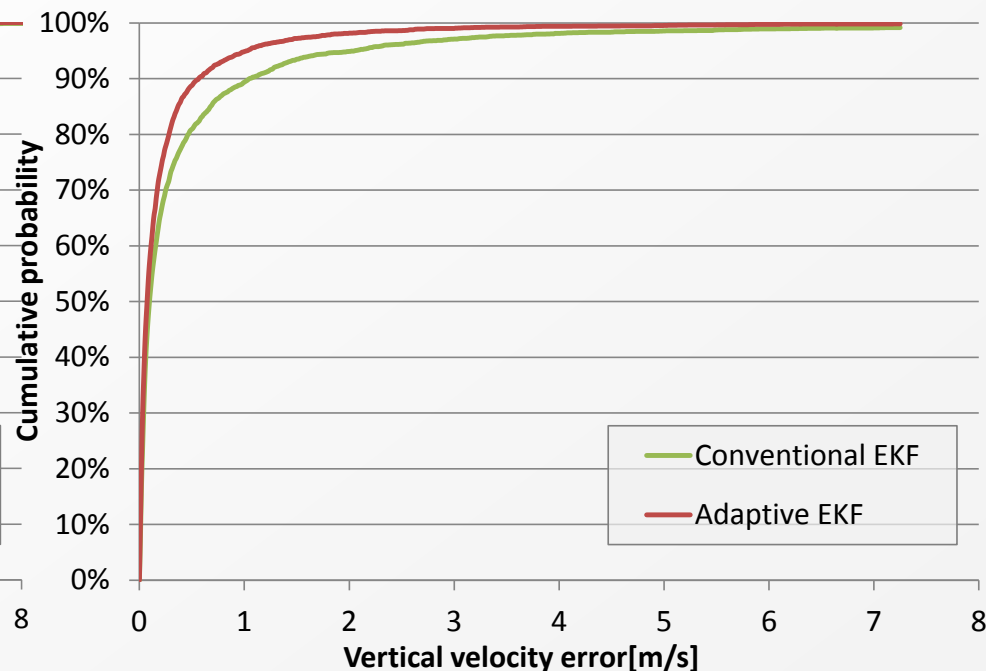
# Velocity error



## Horizontal CDF



## Vertical CDF



		Mean error[m/s]	Error at 68.27% [m/s]	Error at 95.45% [m/s]	Error at 100% [m/s]
Horizontal	Adaptive EKF	0.010	0.12	0.73	6.92
	Conventional EKF	0.027	0.16	1.39	15.80
Vertical	Adaptive EKF	0.033	0.16	1.07	11.63
	Conventional EKF	0.146	0.23	2.19	-16.91

- The adaptive EKF achieved the impressive GNSS performance using the mass-product receiver in the dense urban environment.
  - The positioning accuracy and precision are drastically improved comparing with the conventional EKF.

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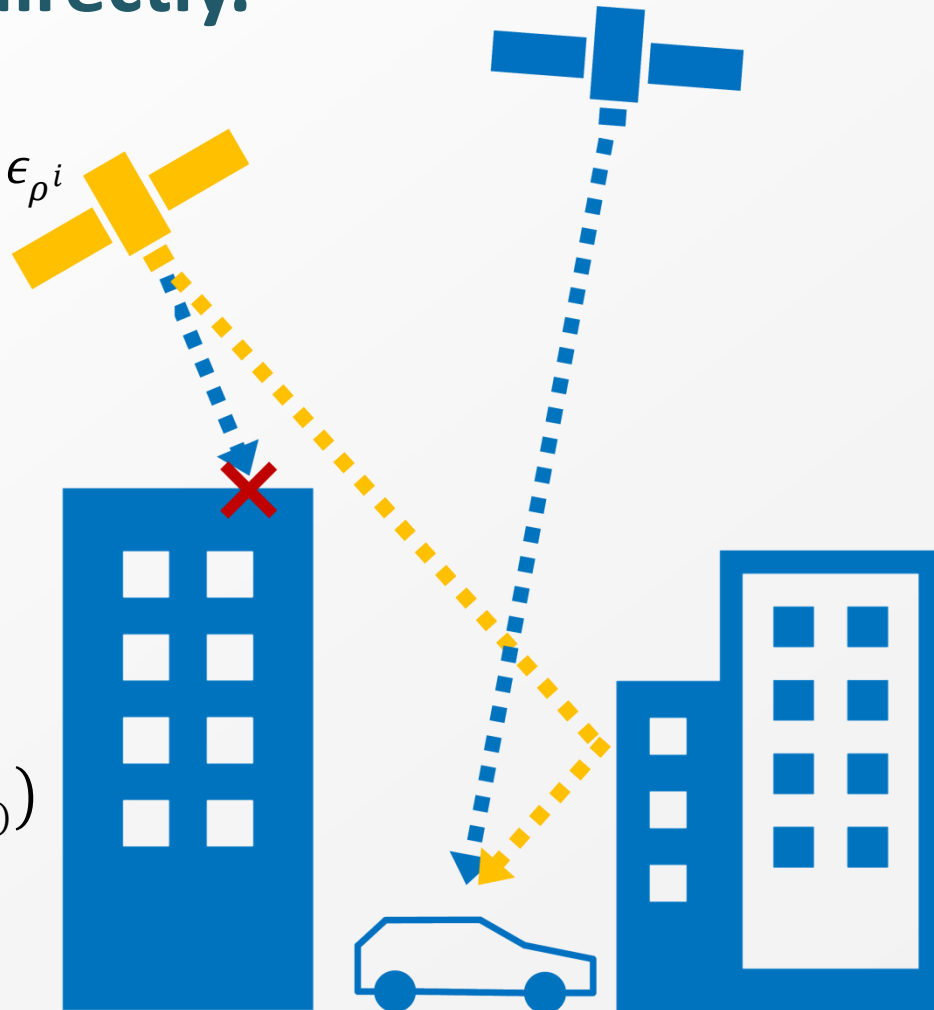
- Residuals of single difference gives NLOS bias directly.

$$\rho^i = \gamma^i + \delta t + \boxed{\delta_{NLOS}^i} + \epsilon_{\rho^i}$$

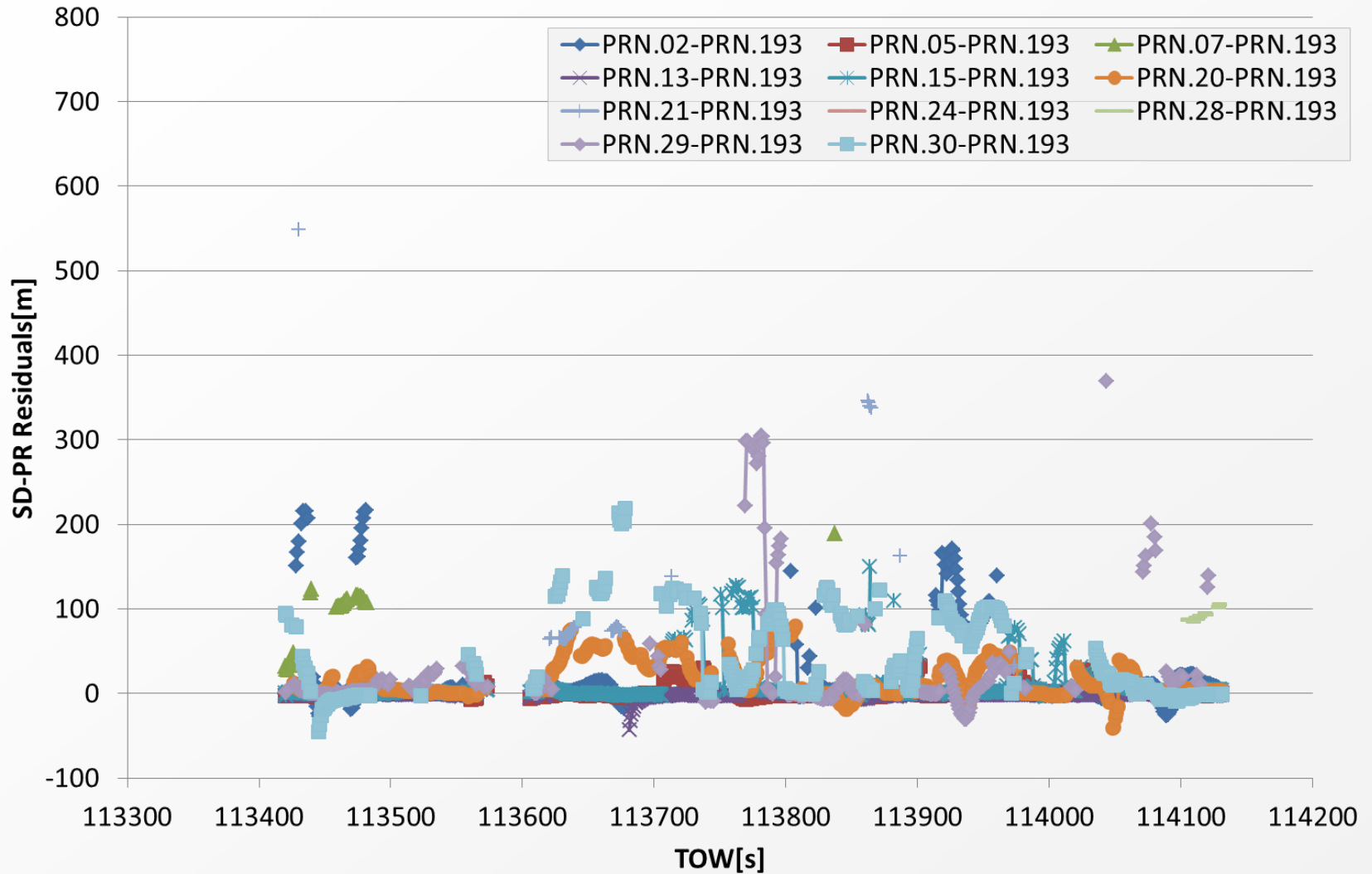
$$\rho^{QZSS} = \gamma^{QZSS} + \delta t + \epsilon_{\rho^{QZSS}}$$



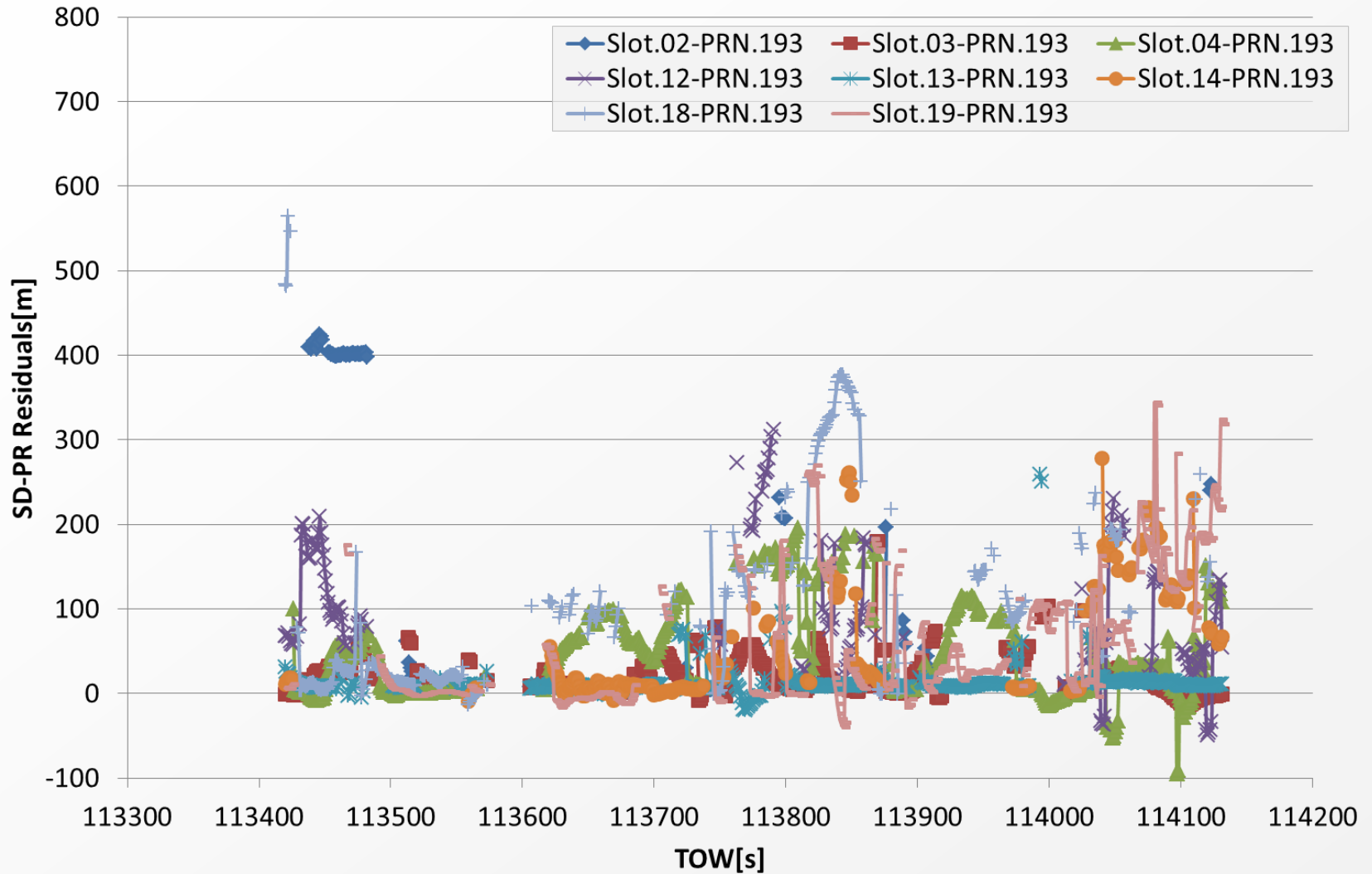
$$\begin{aligned} \Delta\rho_{(residual)}^{QZSS,i} &= (\rho^i - \rho^{QZSS}) \\ &\quad - (\gamma_{(true)}^i - \gamma_{(true)}^{QZSS}) \\ &= \boxed{\delta_{NLOS}^i} + \epsilon_{\Delta\rho^{QZSS,i}} \end{aligned}$$



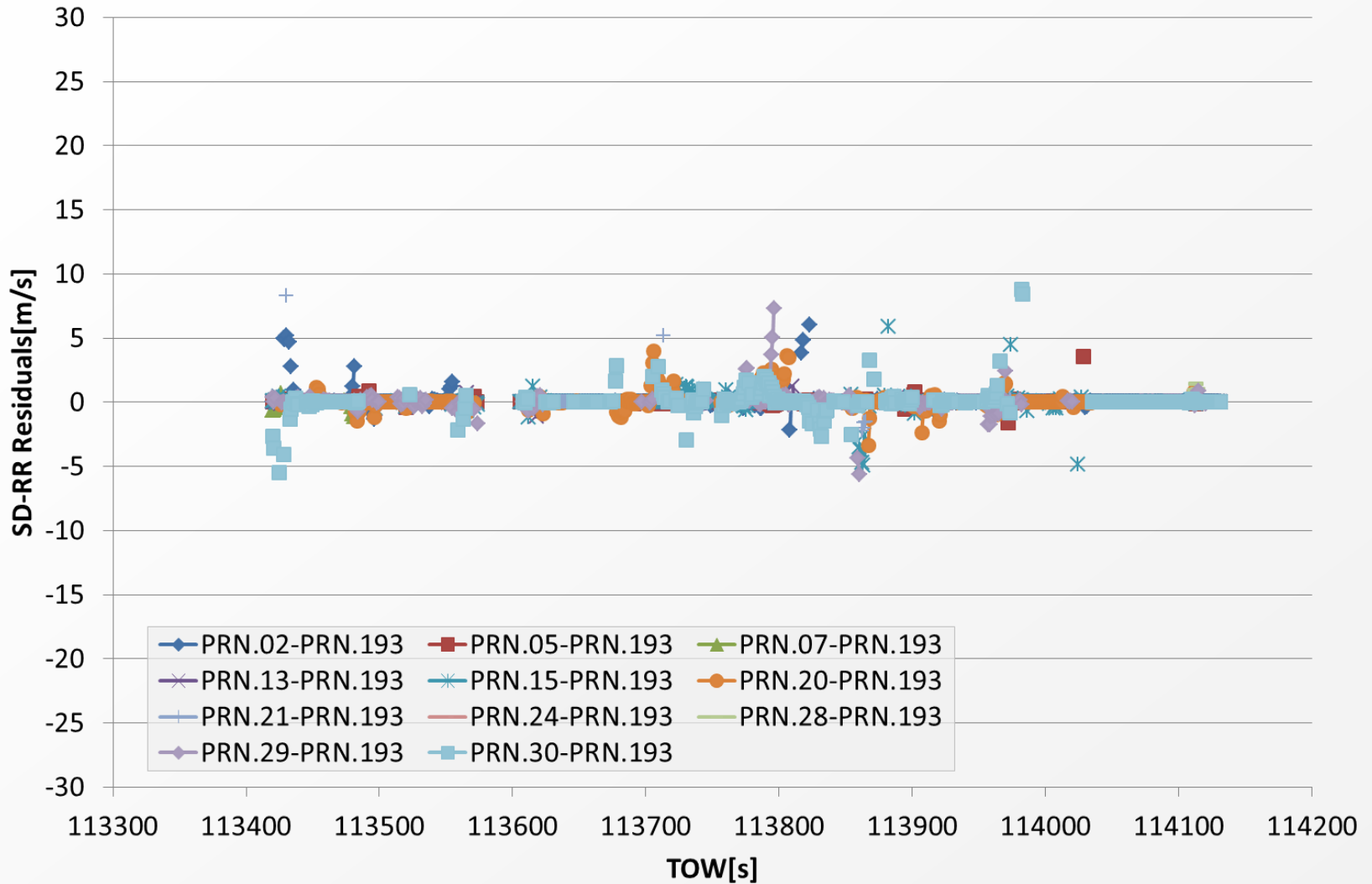
# GPS PR error (lap04)



# GLO PR error (lap04)

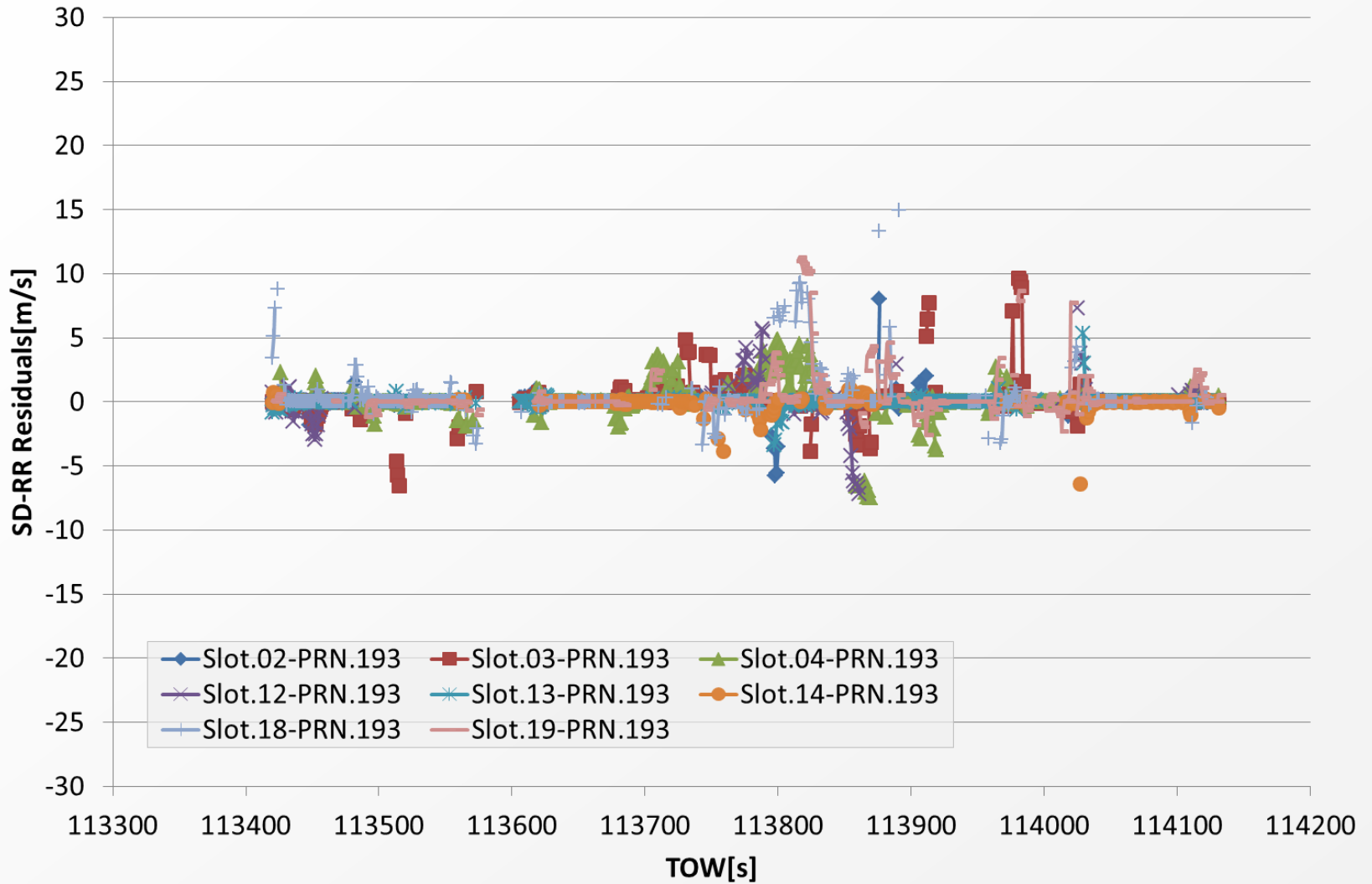


# GPS RR error (lap04)

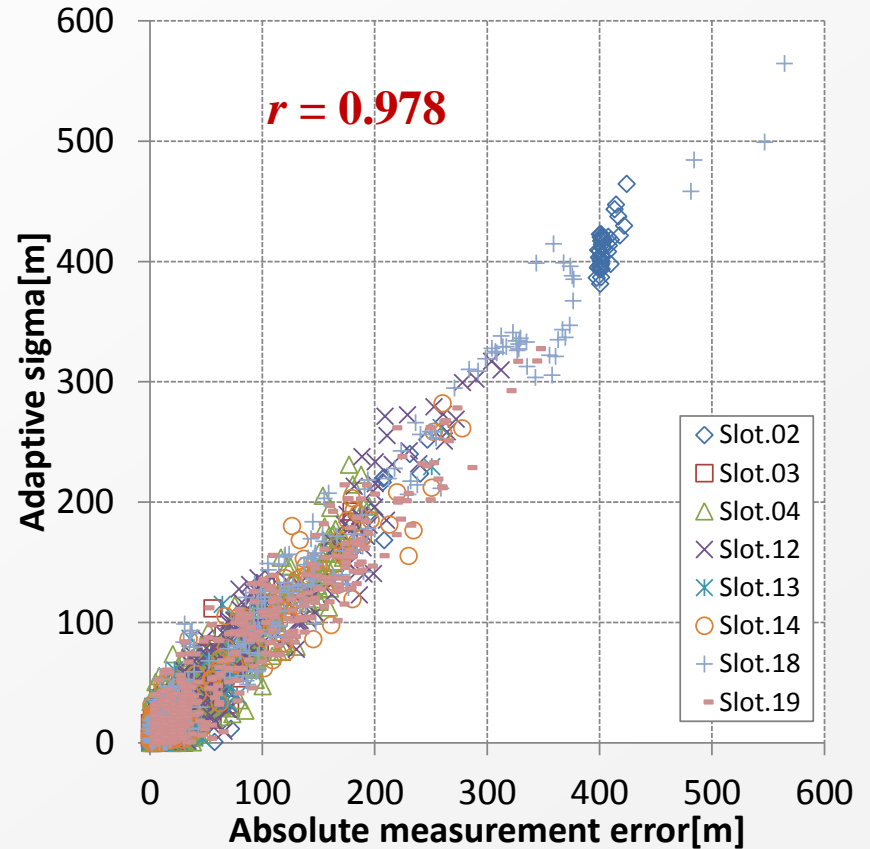
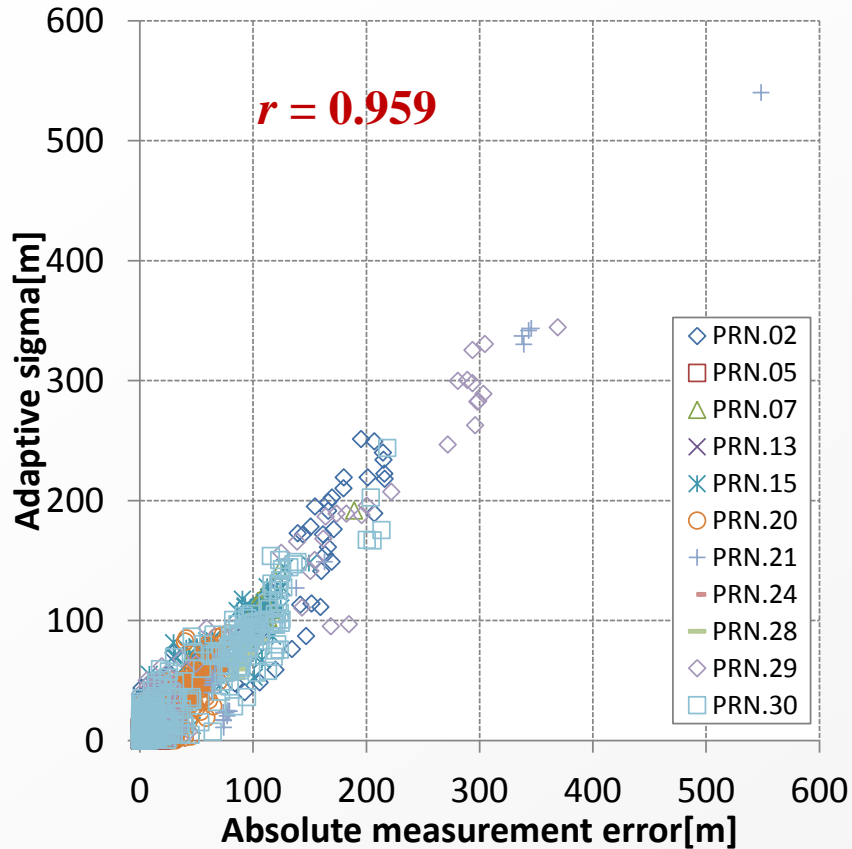




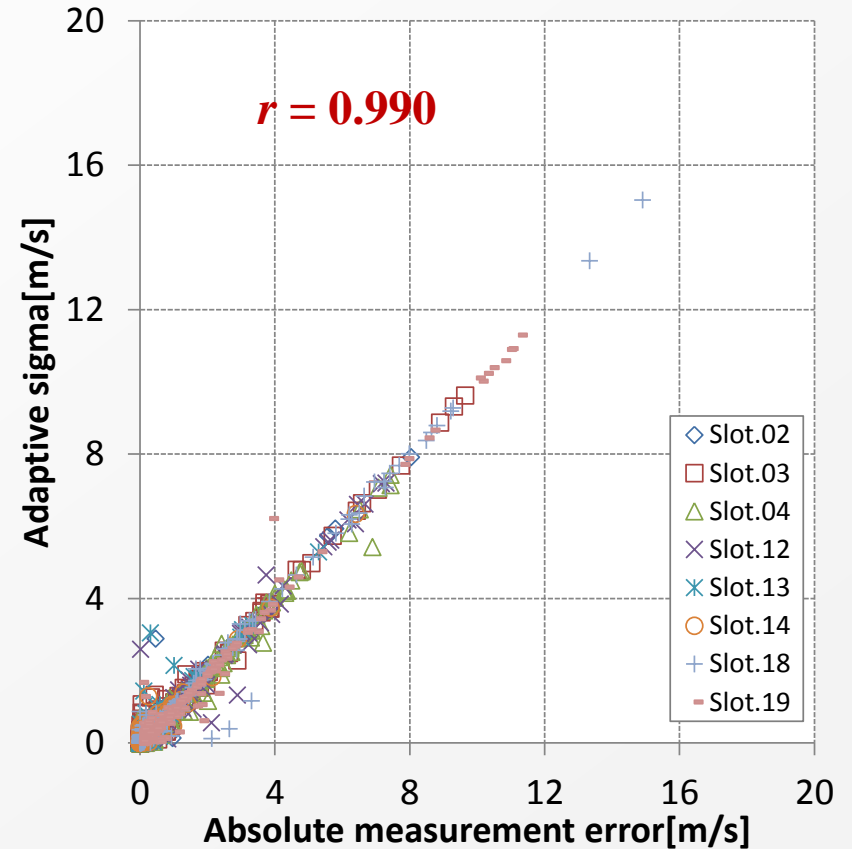
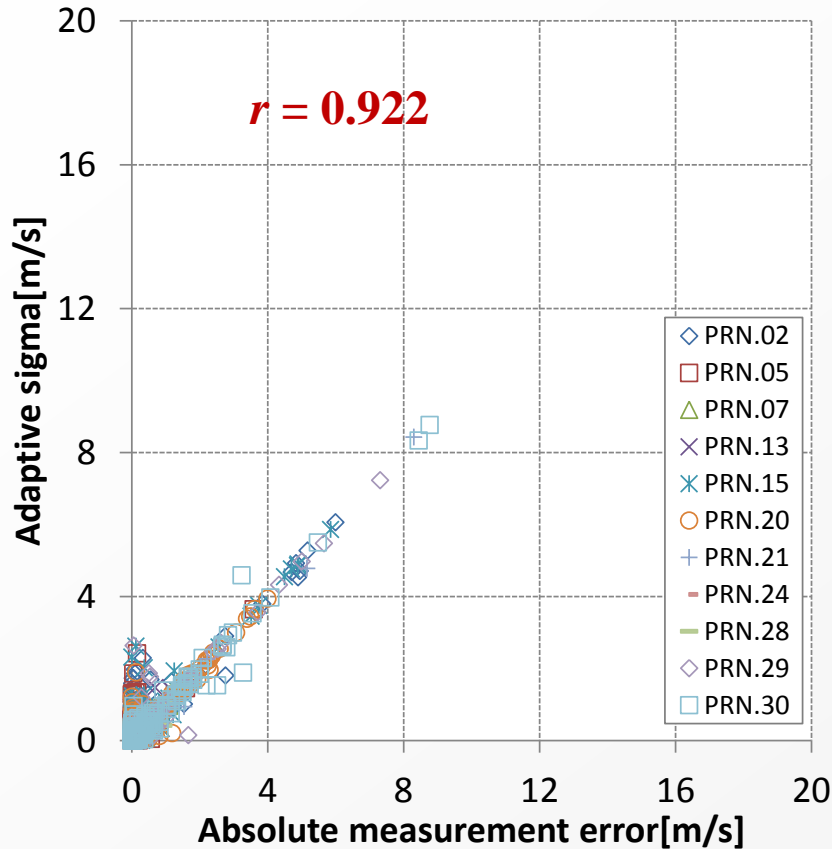
# GLO RR error (lap04)



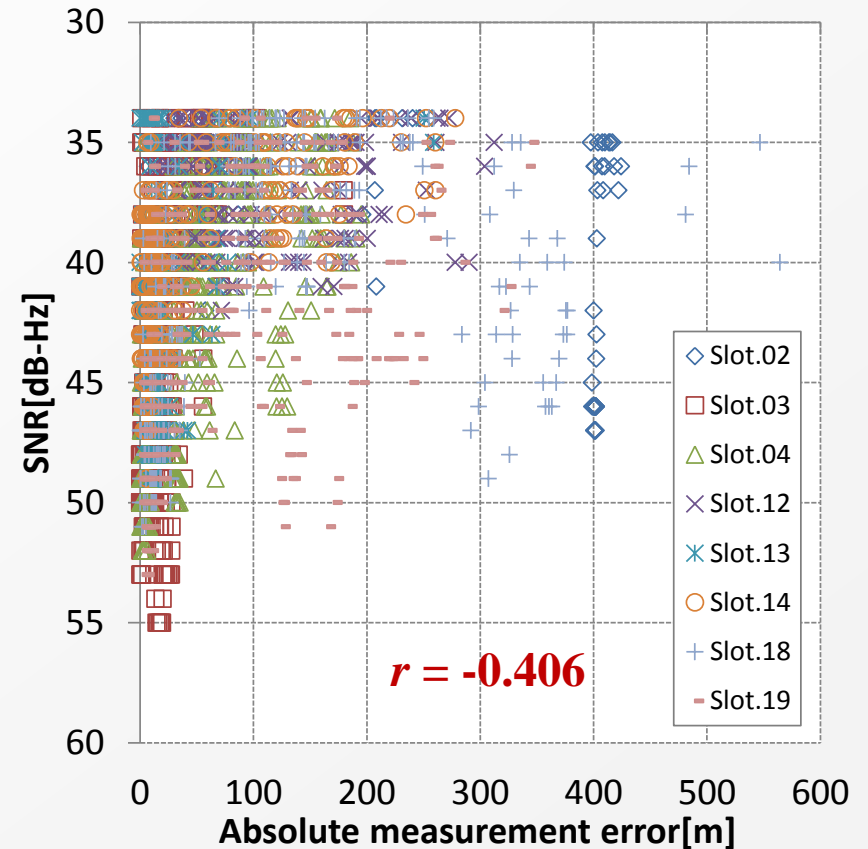
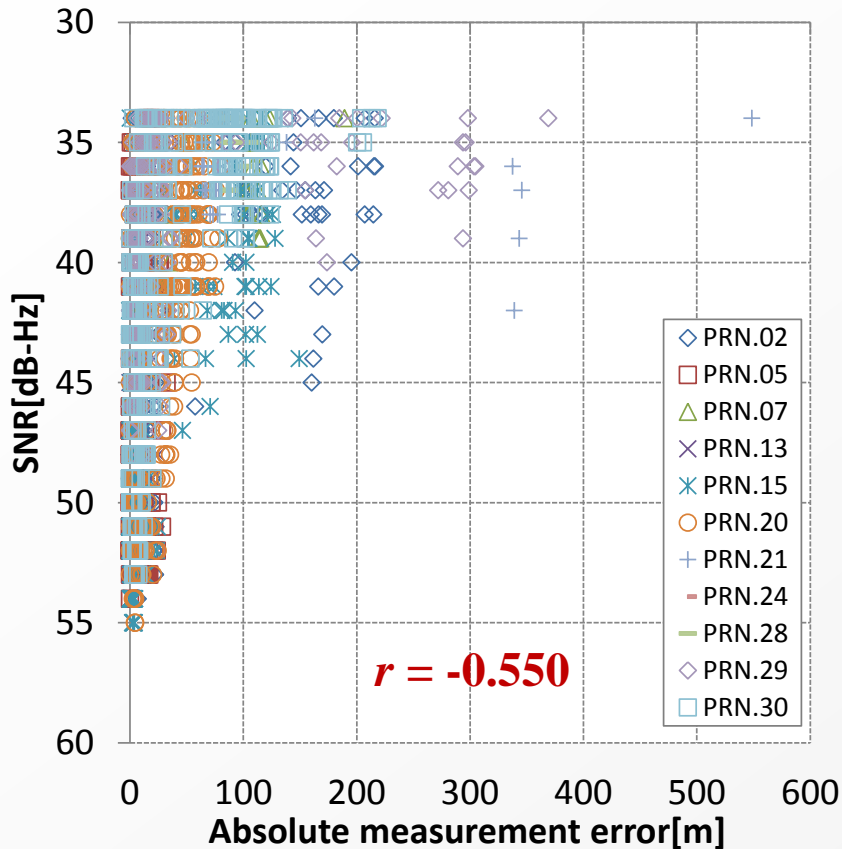
# GPS PR error vs Adaptive $\sigma$ (lap04)



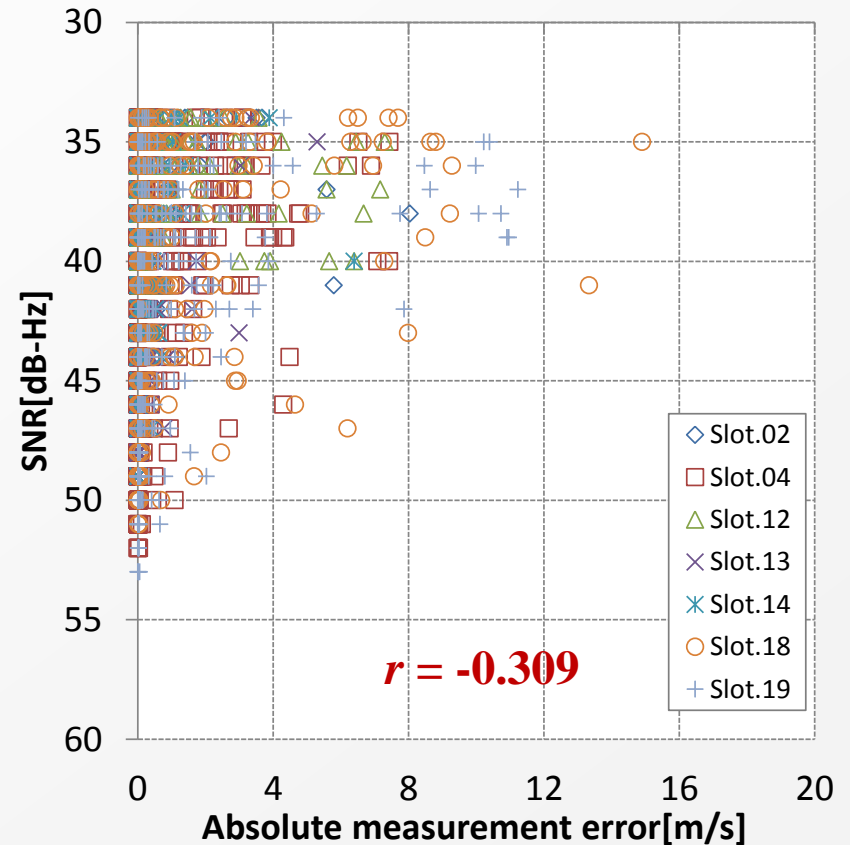
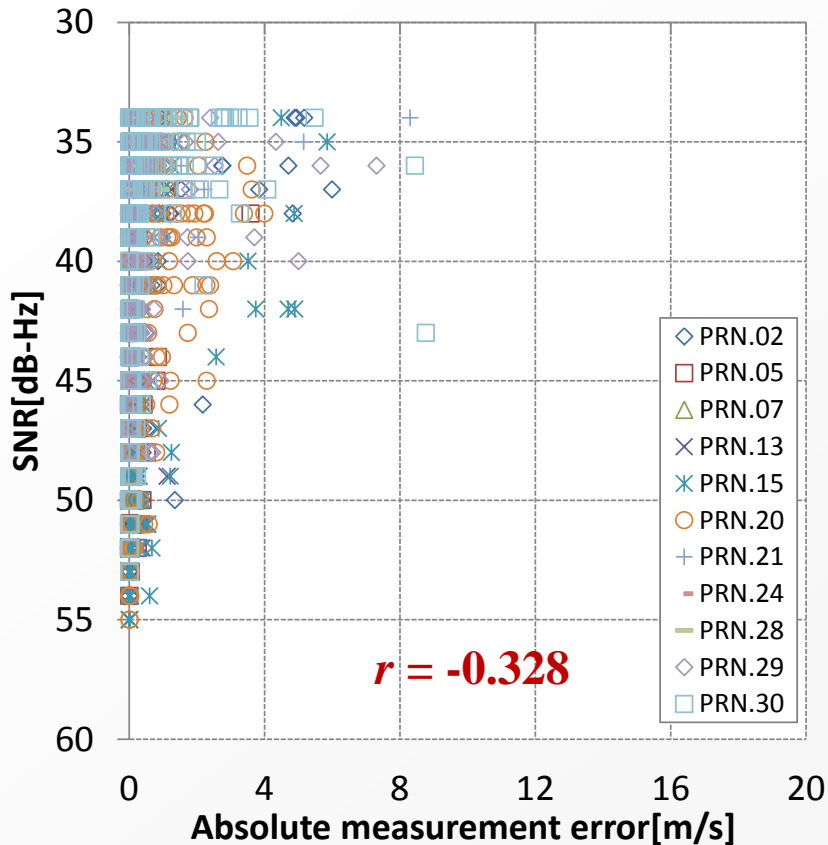
# GLO PR error vs Adaptive $\sigma$ (lap04)



# PR error vs SNR (lap04)



# RR error vs SNR (lap04)



- **Large NLOS bias can be found in both pseudo-range and Doppler measurements.**
- **Adaptive  $\sigma$  matched the NLOS bias well.**
  - While it was the challenge for conventional SNR-based  $\sigma$  estimation.
  - This is the exact reason why the single point positioning performance by the Adaptive EKF improved.

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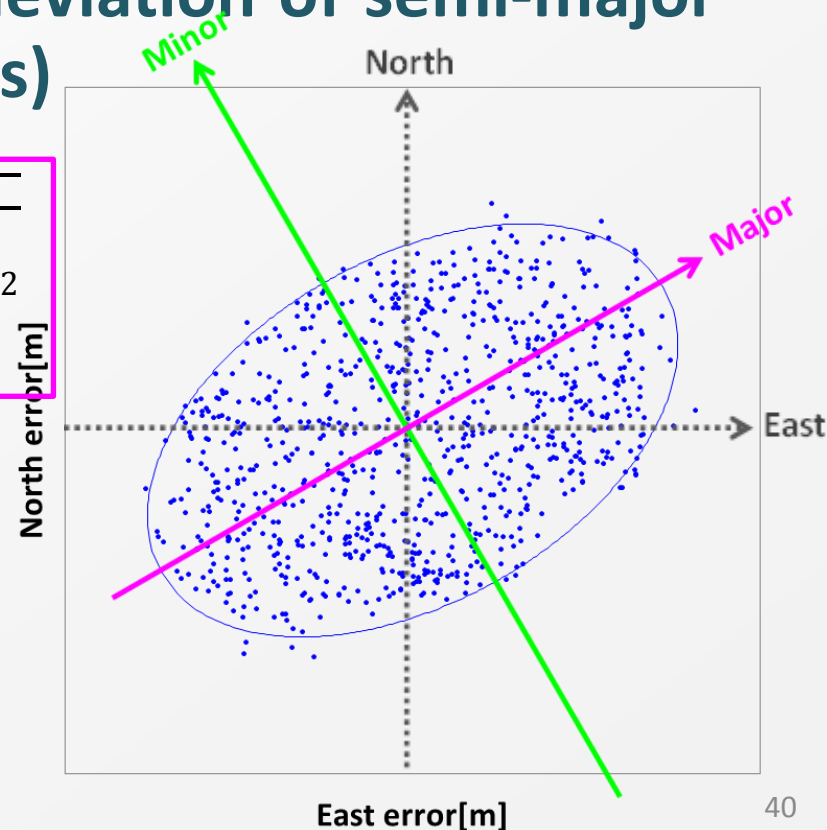
- RTCA defines HPL (Horizontal Protection Level) and VPL (Vertical Protection Level) regarding to standard deviations.
  - Error ellipsoid: Standard deviation of semi-major axis of error ellipse (meters)

$$\sigma_{Hmajor} = \sqrt{\frac{\sigma_e^2 + \sigma_n^2}{2} + \sqrt{\left(\frac{\sigma_e^2 - \sigma_n^2}{2}\right)^2 + \sigma_{en}^2}}$$

$$HPL = k \cdot \sigma_{Hmajor}$$

$$VPL = k \cdot \sigma_u$$

- RTCA suggests  $k=6$ .





- Rotate the covariance matrix of the state vector from ECEF to ENU coordinates frame.

$$\begin{aligned} \mathbf{P}_{g_u,ENU} &= \mathbf{T}^T \mathbf{P}_{g_u} \mathbf{T} \\ &= \begin{pmatrix} \sigma_e^2 & \sigma_{en} & \sigma_{eu} \\ \sigma_{ne} & \sigma_n^2 & \sigma_{nu} \\ \sigma_{ue} & \sigma_{un} & \sigma_u^2 \end{pmatrix} \end{aligned}$$

- where  $\mathbf{T}$  is a rotation matrix from ECEF to ENU coordinates.
- This rotation serves HPL and VPL computation.

- Summary for extended-Kalman filter (EKF)

Prediction  
(Time update)

$$\begin{aligned} \mathbf{x}_{\bar{k}} &= \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{D}_k \mathbf{u}_k \\ \mathbf{P}_{\bar{k}} &= \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_{k-1}^T + \mathbf{Q}_k \end{aligned}$$

$$\begin{aligned} \mathbf{K}_k &= \frac{\mathbf{P}_{\bar{k}} \mathbf{H}(\mathbf{x}_{\bar{k}})^T}{\left( \mathbf{H}(\mathbf{x}_{\bar{k}}) \mathbf{P}_{\bar{k}} \mathbf{H}(\mathbf{x}_{\bar{k}})^T + \widehat{\mathbf{R}}_k \right)} \\ \mathbf{x}_k &= \mathbf{x}_{\bar{k}} + \mathbf{K}_k (\mathbf{z}_k - \mathbf{h}(\mathbf{x}_{\bar{k}})) \\ \mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}(\mathbf{x}_{\bar{k}})) \mathbf{P}_{\bar{k}} \end{aligned}$$

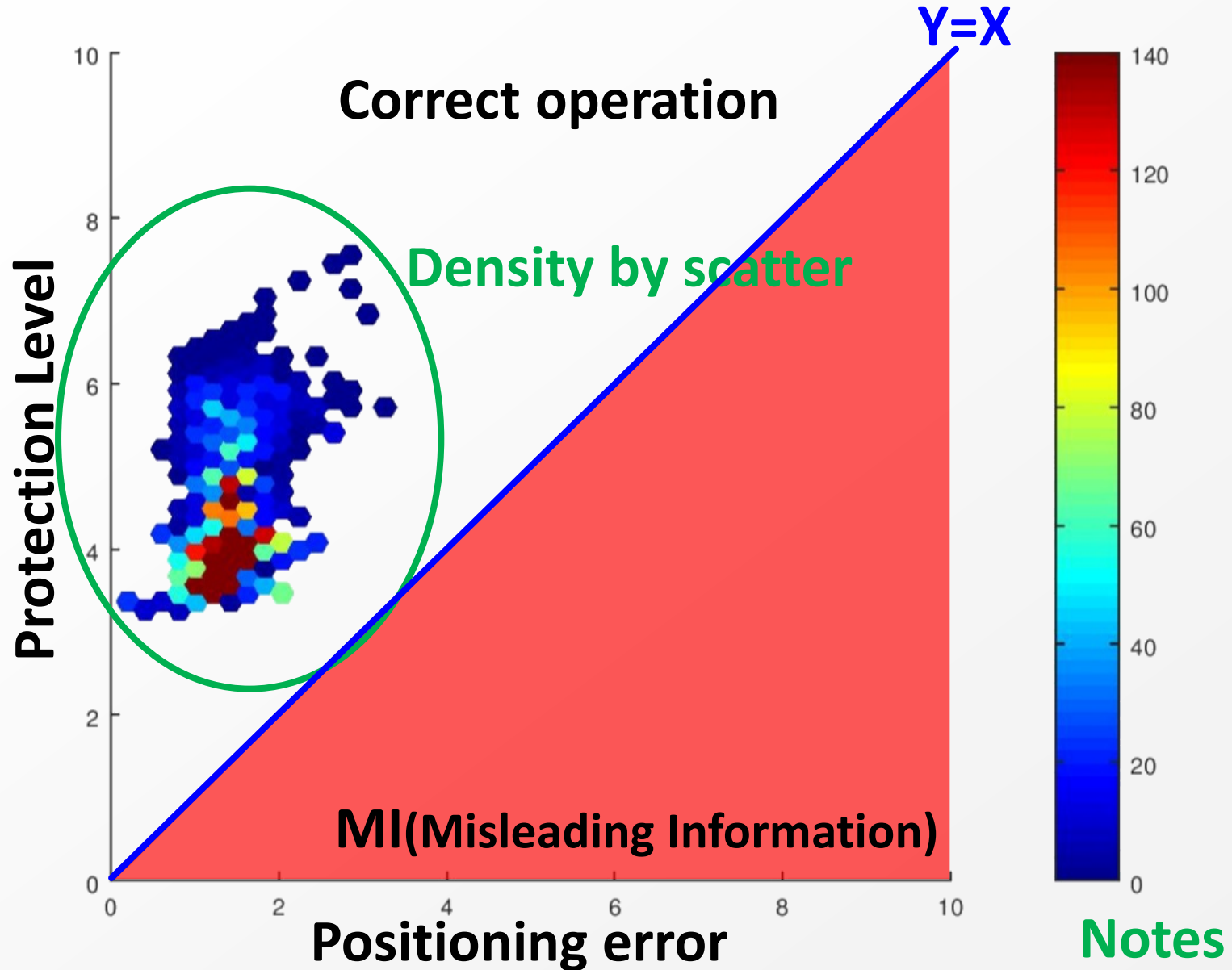
Filter  
(Measurement Update)

Measurement

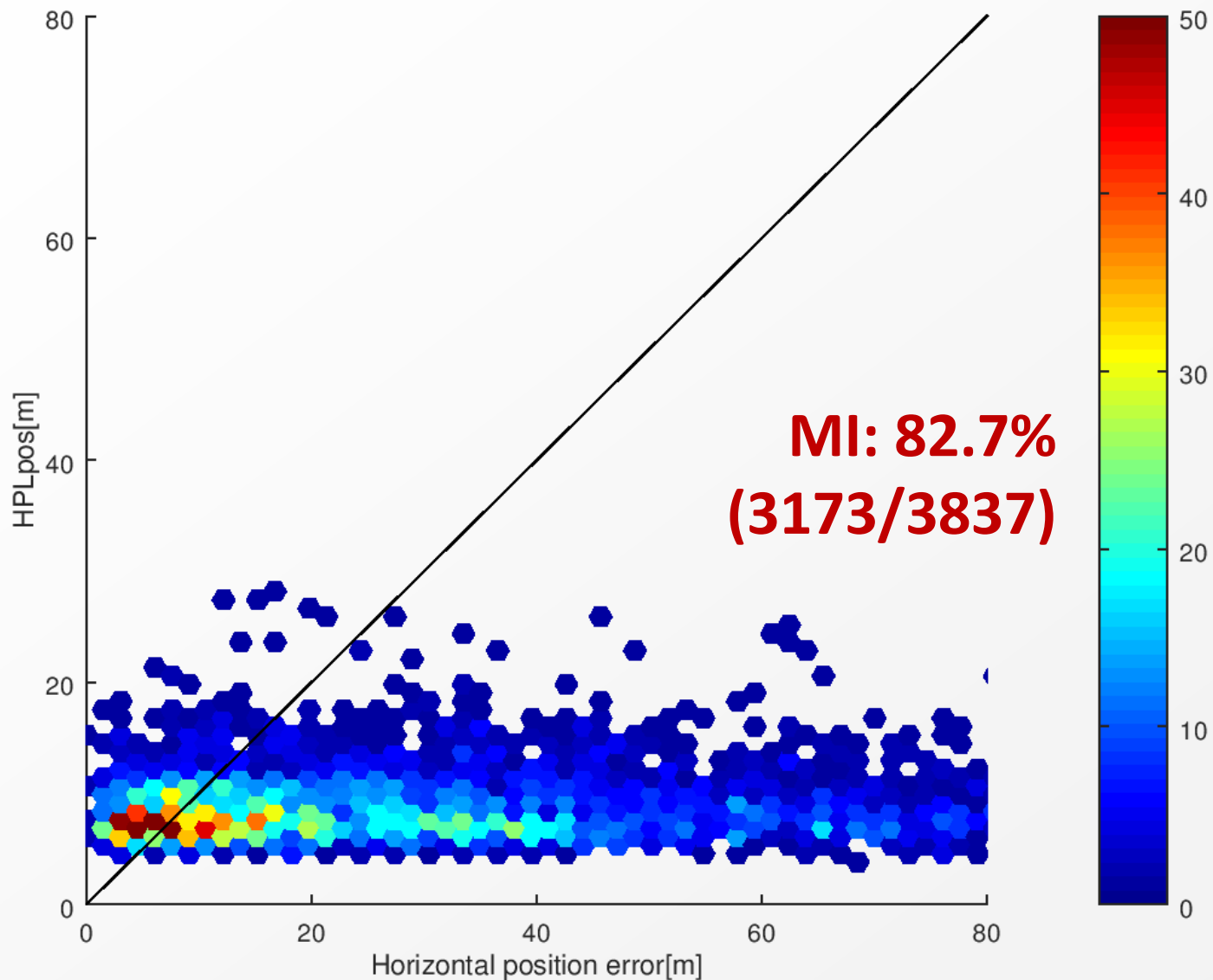
$$\begin{aligned} \mathbf{z}_k \\ \widehat{\mathbf{R}}_k \end{aligned}$$

Adaptive covariance matrix  
of measurement noise

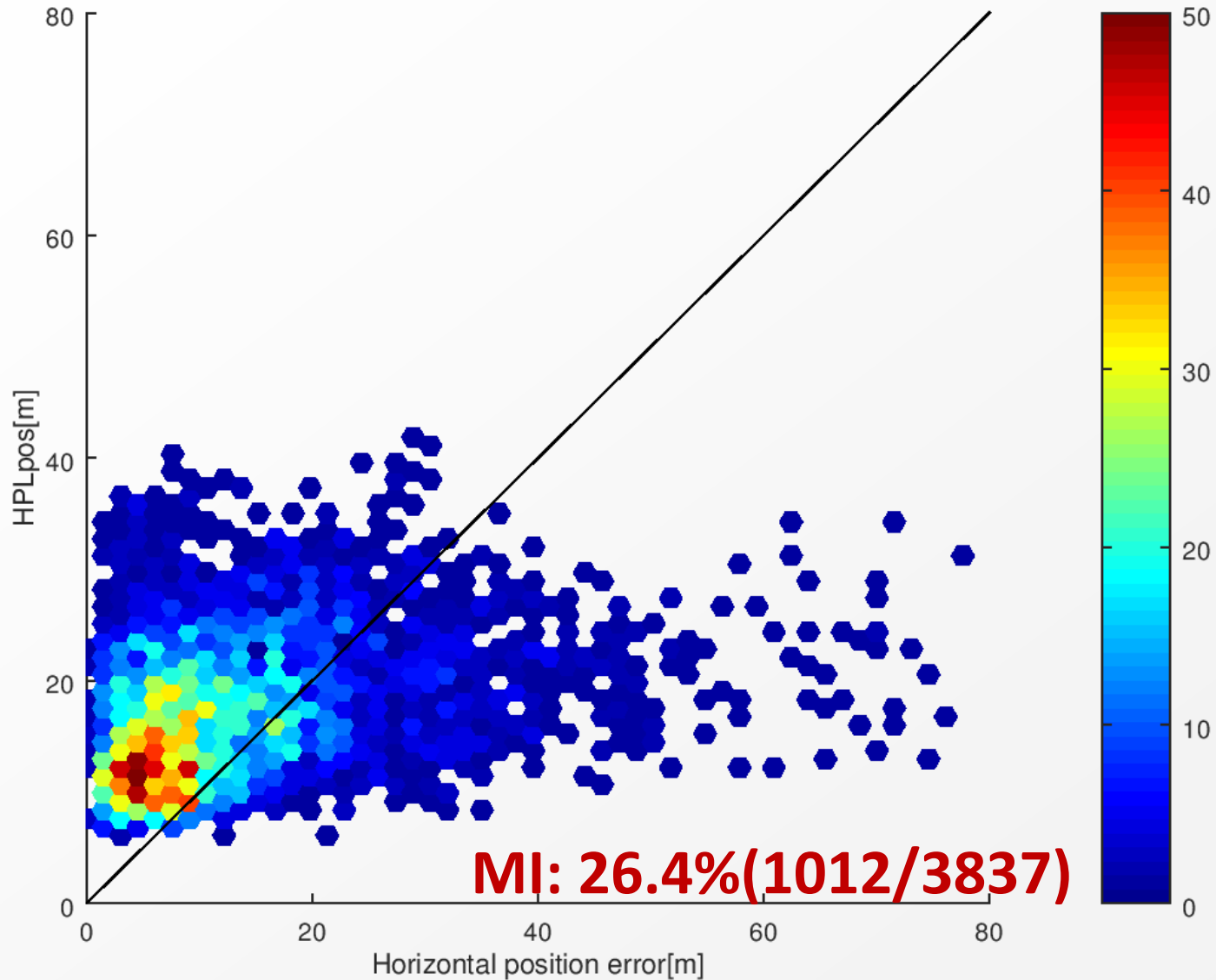
epoch  $k$



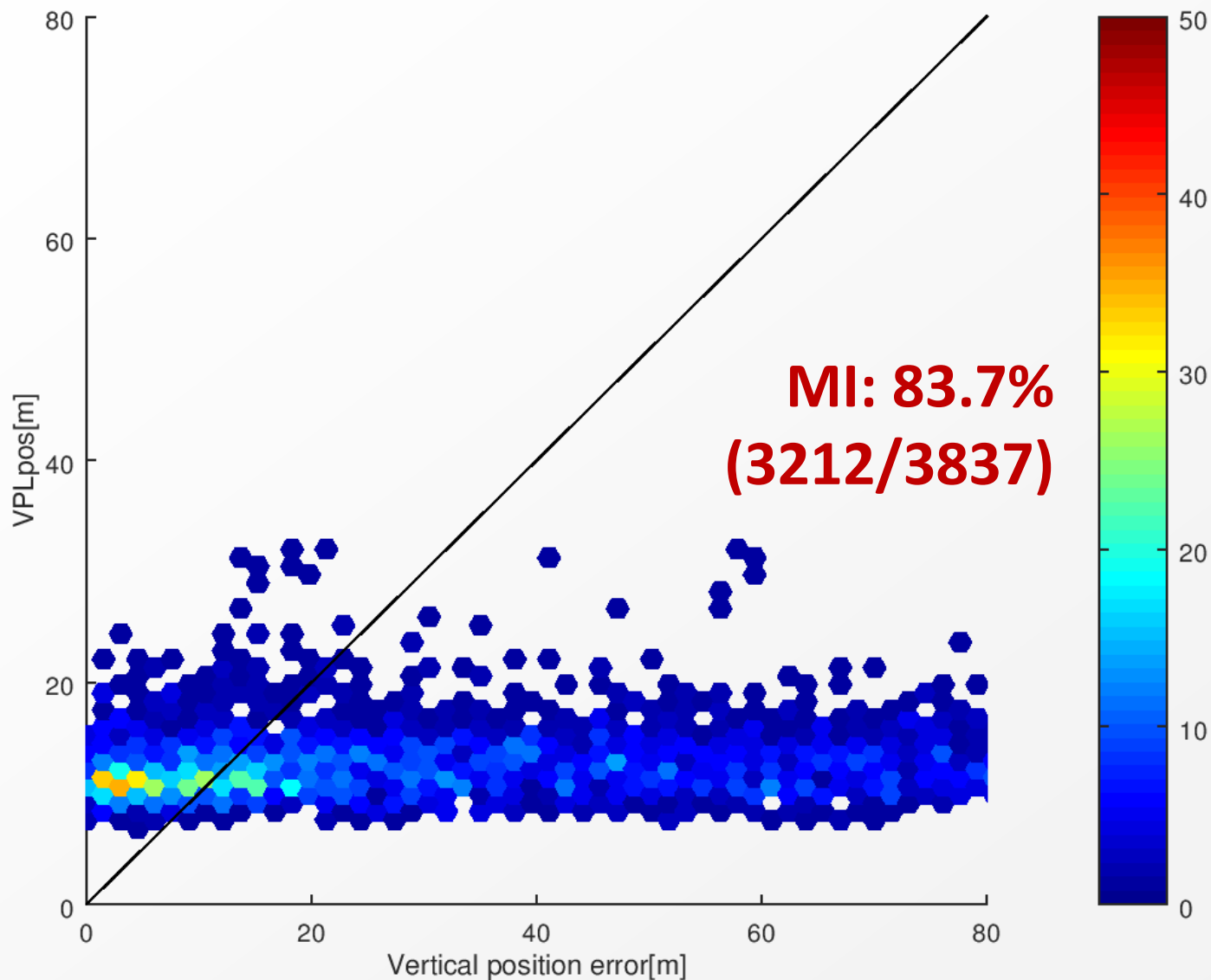
# HPLpos by Conventional EKF



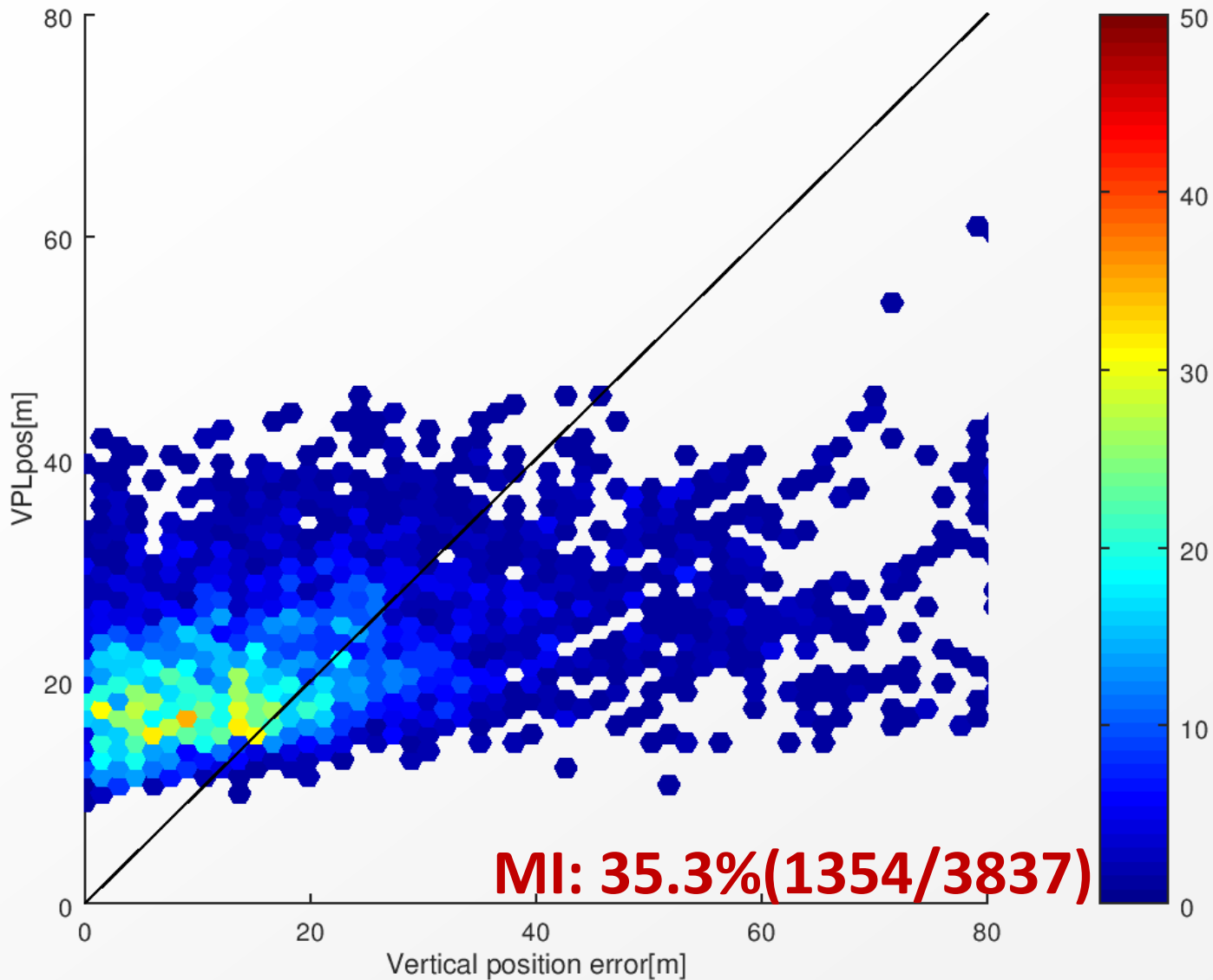
# HPLpos by Adaptive EKF



# VPLpos by Conventional EKF



# VPLpos by Adaptive EKF



- **Protection Levels by the conventional EKF degraded.**
  - No longer the integrity information.
- **Adaptive EKF restrained the degradation.**
  - MI stood at the marginal.



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- **The adaptive EKF achieved the impressive GNSS performance using the mass-product receiver in the dense urban environment:**
  - The positioning accuracy and precision are drastically improved comparing with the conventional EKF.
  - Adapted covariance matrix matched the actual measurement errors well,
    - It was the challenge for the conventional SNR-based estimation.
  - Integrity information degraded by the conventional EKF, while adaptive EKF restrained the degradation.

- **The ideal MI (Miss-leading Information) is 0%.**
  - Further investigation and improvement are necessary to establish more robust integrity information.
  - Alternate detection methods of outliers must be considered:
    - Residual-based test
    - Solution separation

- **The author would like to thank the colleagues of:**
  - **Furuno Electric Co,. Ltd.**
  - **eRide, Inc.**
    - **building the test environment**
    - **collecting test drive data**

**Thank you very much  
for your kind attention!**

- **GNSS pseudo-range measurement can be modeled by simply:**

$$\rho^i = \gamma^i + \delta t + \epsilon_{\rho^i}$$

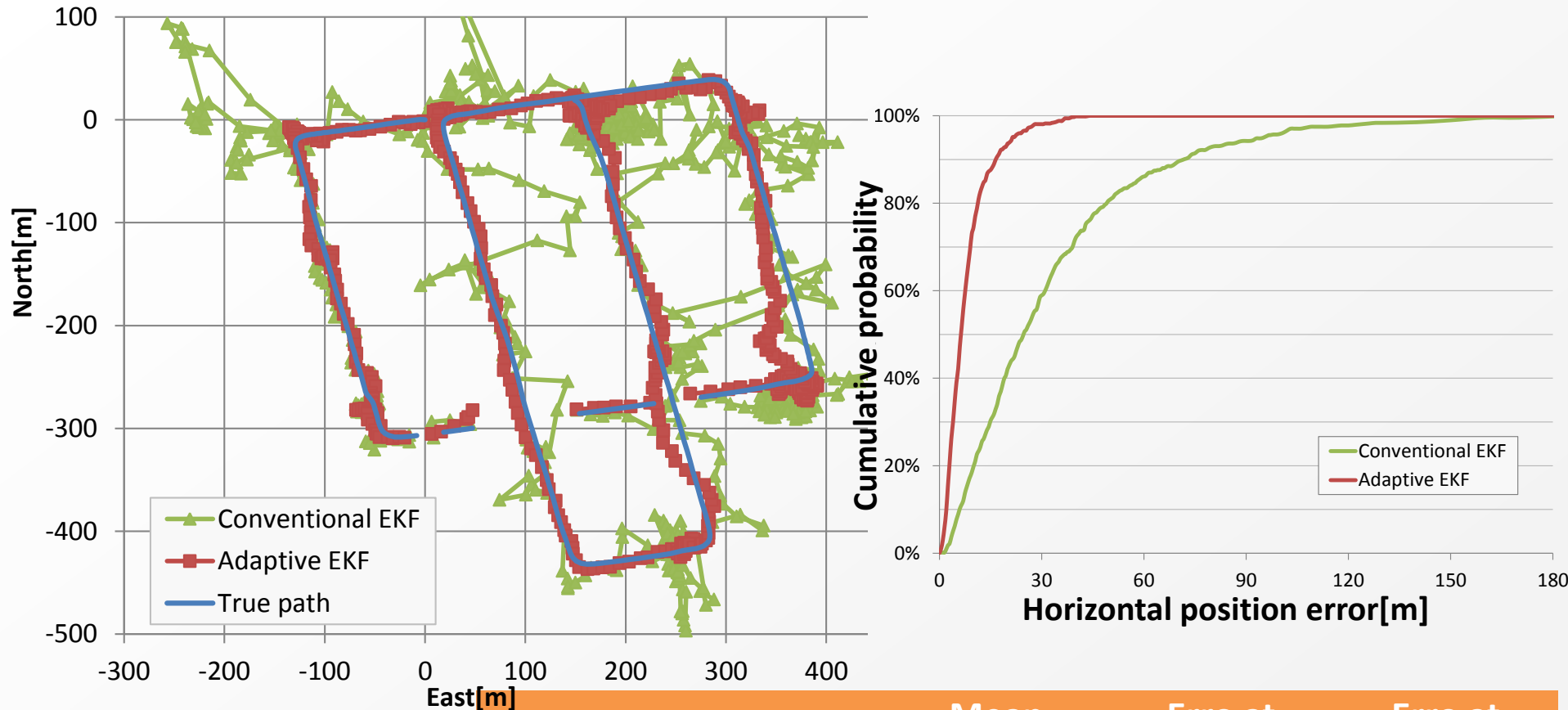
$$\gamma^i = \|\mathbf{g}^i - \mathbf{g}_u\|$$

$$= \sqrt{(x^i - x_u)^2 + (y^i - y_u)^2 + (z^i - z_u)^2}$$

- **Then, linearize by Taylor series:**

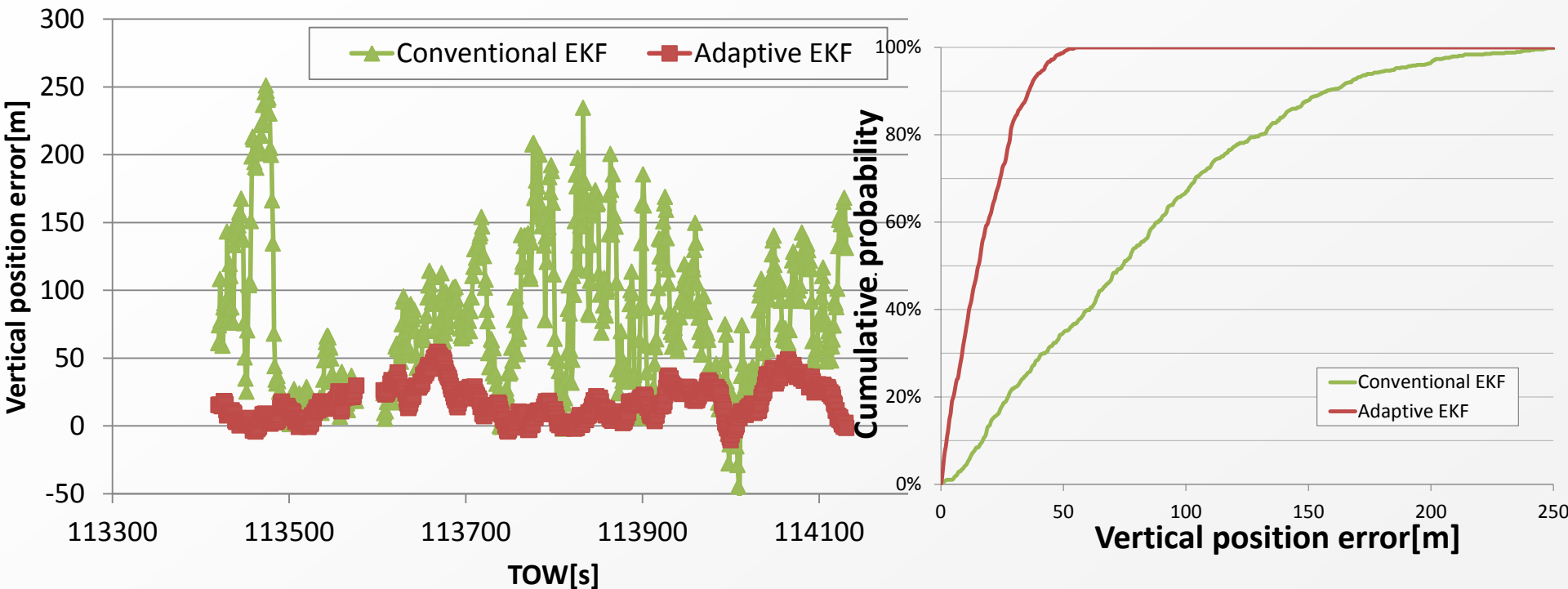
$$\begin{aligned} \rho^i \sim & \left( \sqrt{(x^i - x_0)^2 + (y^i - y_0)^2 + (z^i - z_0)^2} + \delta t_0 \right) \\ & + \frac{(x^i - x_0)\Delta x + (y^i - y_0)\Delta y + (z^i - z_0)\Delta z}{\sqrt{(x^i - x_0)^2 + (y^i - y_0)^2 + (z^i - z_0)^2}} + \Delta \delta t + \epsilon_{\rho^i} \end{aligned}$$

# Horizontal pos. error (lap04)



	Mean error[m]	Erro at 68.27%[m]	Erro at 95.45%[m]
<b>Adaptive EKF</b>	<b>1.58</b>	<b>8.9</b>	<b>22.5</b>
<b>Conventional EKF</b>	<b>7.38</b>	<b>36.8</b>	<b>96.6</b>

# Vertical pos. error (lap04)



	Mean error[m]	Erro at 68.27%[m]	Erro at 95.45%[m]
<b>Adaptive EKF</b>	<b>17.16</b>	<b>23.6</b>	<b>42.5</b>
<b>Conventional EKF</b>	<b>80.43</b>	<b>102.0</b>	<b>180.4</b>



# PR NLOS error

	GPS/QZSS			GLONASS		
	Mean [m/s]	StDev [m/s]	Max [m/s]	Mean [m/s]	StDev [m/s]	Max [m/s]
Lap01	32.9	66.9	593.4	20.2	41.6	297.5
Lap02	43.1	94.0	530.8	24.6	48.5	477.4
Lap03	33.5	77.8	491.8	50.5	87.3	516.9
Lap04	14.0	38.6	548.6	53.4	82.5	564.5
Lap05	34.8	83.6	542.8	33.9	77.2	731.3
Lap06	33.9	84.4	576.1	21.6	55.8	531.7

# Doppler NLOS error

	GPS/QZSS			GLONASS		
	Mean [m/s]	StDev [m/s]	Max [m/s]	Mean [m/s]	StDev [m/s]	Max [m/s]
Lap01	0.06	1.74	-23.11	0.05	1.15	-14.97
Lap02	0.27	2.03	16.90	0.14	1.24	9.92
Lap03	0.15	1.28	23.74	0.44	2.57	24.68
Lap04	0.04	0.59	8.77	0.20	1.44	14.90
Lap05	0.04	1.19	-18.27	0.12	1.58	23.06
Lap06	0.20	1.77	20.15	0.09	1.12	16.80

# Adaptive $\sigma$ vs. meas error

	GPS/QZSS		GLONASS	
	Pseudo-range	Doppler shift	Pseudo-range	Doppler shift
Lap01	0.932	0.954	0.949	0.976
Lap02	0.990	0.982	0.953	0.954
Lap03	0.983	0.982	0.980	0.996
Lap04	0.959	0.922	0.978	0.990
Lap05	0.979	0.979	0.954	0.987
Lap06	0.989	0.990	0.979	0.989

# SNR vs. meas error

	GPS/QZSS		GLONASS	
	Pseudo-range	Doppler shift	Pseudo-range	Doppler shift
Lap01	-0.517	-0.265	-0.592	-0.276
Lap02	-0.528	-0.357	-0.503	-0.421
Lap03	-0.455	-0.288	-0.521	-0.292
Lap04	-0.550	-0.328	-0.406	-0.309
Lap05	-0.405	-0.232	-0.468	-0.254
Lap06	-0.494	-0.206	-0.549	-0.273

# Stanford diagram

