IPNTJ Summer School on GNSS

B-6 ~ B-8
Theory of Precise Positioning (3)
Advanced Topics

Tokyo Univ. of Marine Science and Technology
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2013-08-19 ~ 24 @Tokyo, Japan

Timetable

B-1	Introduction of RTKLIB	Aug 20	8:50-10:10
B-2	Practice of RTKLIB		10:30-11:50
B-3	Theory of Precise Positioning (1)		13:00-14:20
B-4	Theory of Precise Positioning (2)		14:40-16:00
B-5	Practice of RTK		16:20-17:40
B-6	**Theory of Precise Positioning (3)**	Aug 21	8:50-10:10
B-7	**Practice of PPP**		10:30-11:50
B-8	**Advanced Topics**		13:00-14:20
B-9	**JAXA Activities by Tateshita-san**		14:40-16:00
B-10	**QZSS-Demo**		16:20-17:40

B-6
Theory of Precise Positioning (3)

Time Systems

- **Time Systems**
  - TAI: International Atomic Time
  - UTC: Coordinated Universal Time
  - Local Time (JST, EDT, ...)
  - UT0, UT1, UT2: Universal Time
  - GMST: Greenwich Mean Sidereal Time
  - GPS Time
  - GLONASS Time
  - ...

Time System Conversion

UTC-TAI (s)			
-25	1990/1/1-	-30	1996/1/1-
-26	1991/1/1-	-31	1997/7/1-
-27	1992/7/1-	-32	1999/1/1-
-28	1993/7/1-	-33	2006/1/1-
-29	1994/7/1-	-34	2009/1/1-

TAI to UTC:

$$t_{UTC} = t_{TAI} + \underline{(UTC - TAI)}$$

UTC to UT1:

$$t_{UT1} = t_{UTC} + \underline{(UT1 - UTC)}$$

UT1 to GMST:

$$GMST_{0h\,UT1} = 24110.54841 + 8640184.812866 T'_u + 0.093104 T'^2_u - 6.2\times 10^{-6} T'^3_u$$

$$GMST = GMST_{0h\,UT1} + r(t_{UT1} - t_{0h\,UT1})$$

$$r = 1.002737909350795 + 5.9006\times 10^{-11} T'_u - 5.9\times 10^{-15} T'^2_u$$

$T'_u = d'_u / 36525$ $\quad d'_u$: number of days elapsed since 2000 Jan 1, 12h UT1

GPS Time to TAI:

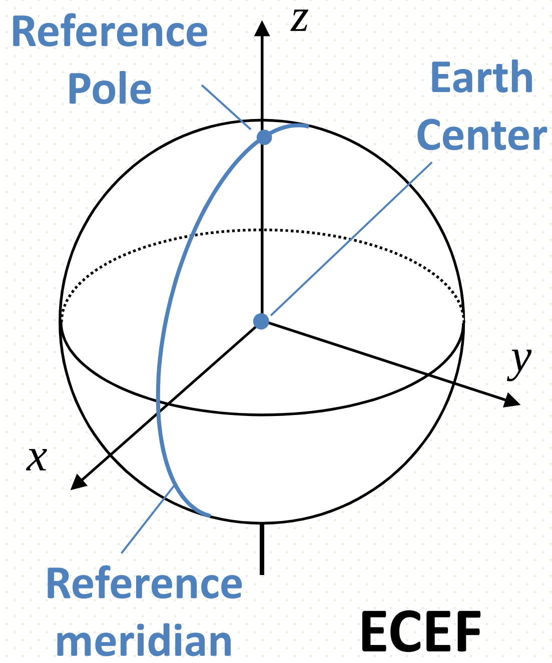
$$t_{TAI} \approx t_{GPST} + 19\,s$$

GPS Time to UTC:

$$t_{UTC} = t_{GPST} - (\underline{\Delta t_{LS}} + \underline{A_0} + \underline{A_1}(t_{GPST} - \underline{t_{ot}}))$$

Coordinate Systems

- **ECEF: Earth-Centered Earth-Fixed**
  - ITRF
  - WGS 84: US (GPS)
  - PZ90: Russia (GLONASS), ...
- **ECI: Earth-Centered Inertial**
  - ICRF: International Celestial Reference Frame
- **ECI-ECEF Connection**
  - Precession/Nutation Model
  - EOP: Earth Orientation Parameters

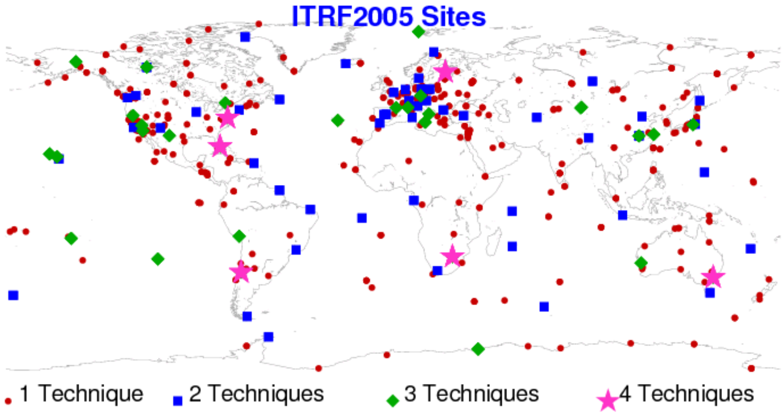


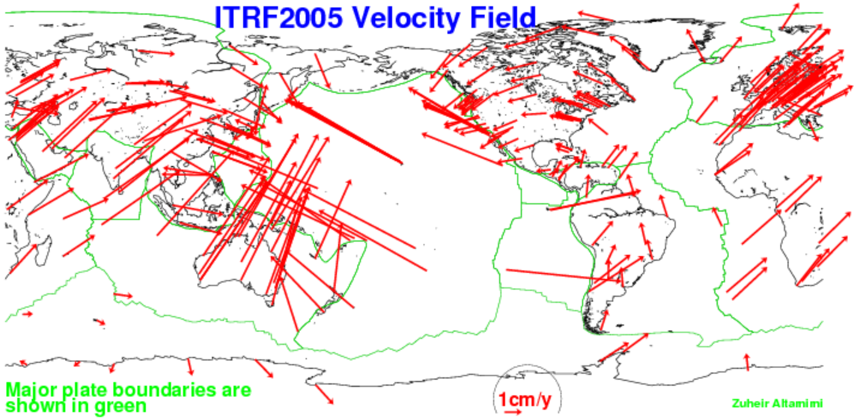
ITRF

- **International Terrestrial Reference Frame**
  – A "Realization" of Maintained by IERS
  – GPS, VLBI, SLR, DORIS Site Position/Velocity List
  – ITRF2005, ITRF2000, ITRF97, ITRF96, ...

VLBI: Very Long Baseline Interferometry
SLR: Satellite Laser Ranging
DORIS: Doppler Orbit determination and Radiopositioning Integrated on Satellite

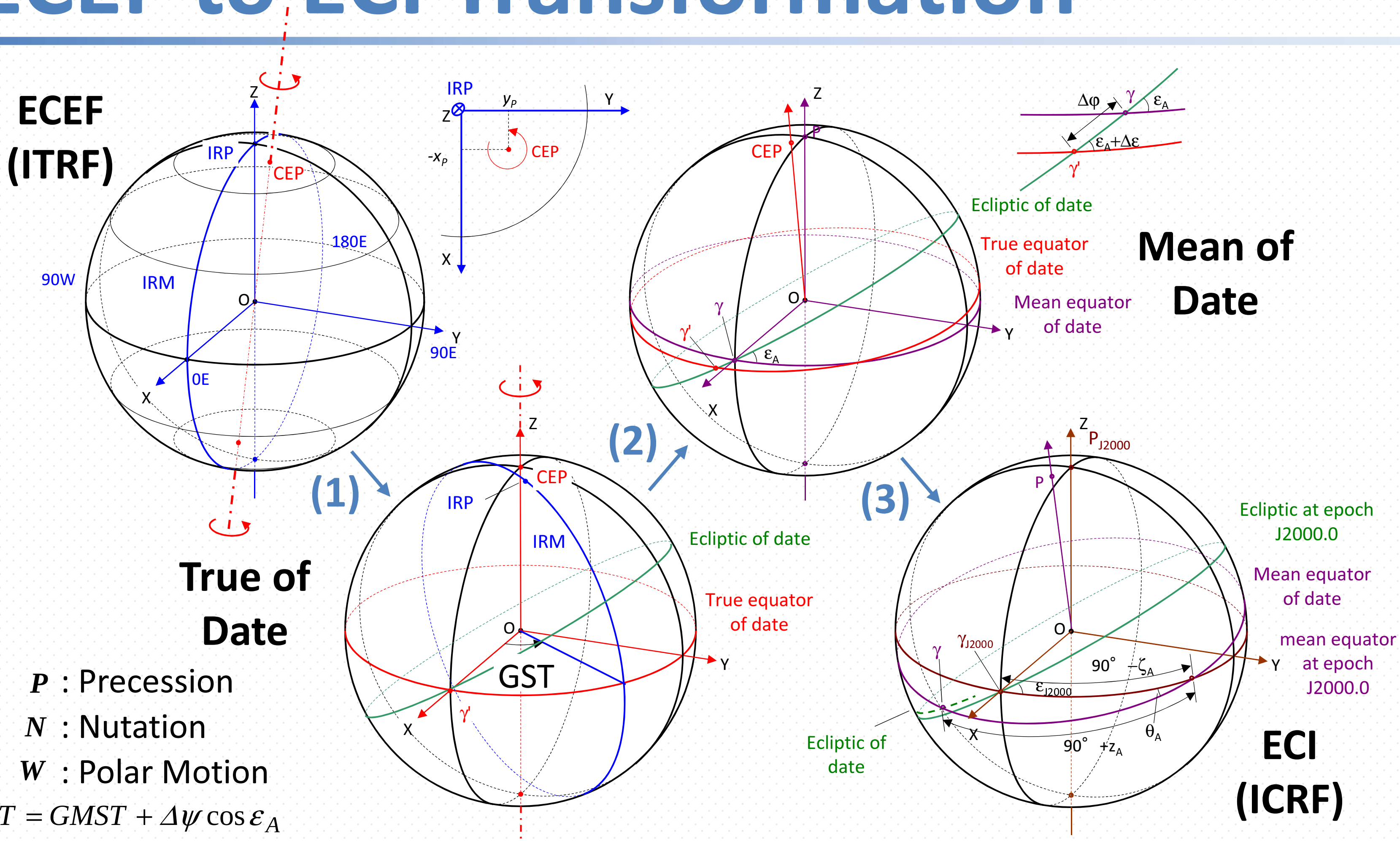
ITRS: International Terrestrial Reference System
IERS: International Earth Rotation Service





http://itrf.ensg.ign.fr/ITRF_solutions/2005/ITRF2005.php

ECEF to ECI Transformation



$\boldsymbol{P}$: Precession

$\boldsymbol{N}$: Nutation

$\boldsymbol{W}$: Polar Motion

$$GST = GMST + \Delta\psi \cos\varepsilon_A + 0''.00264 \sin\Omega + 0''.000063 \sin 2\Omega$$

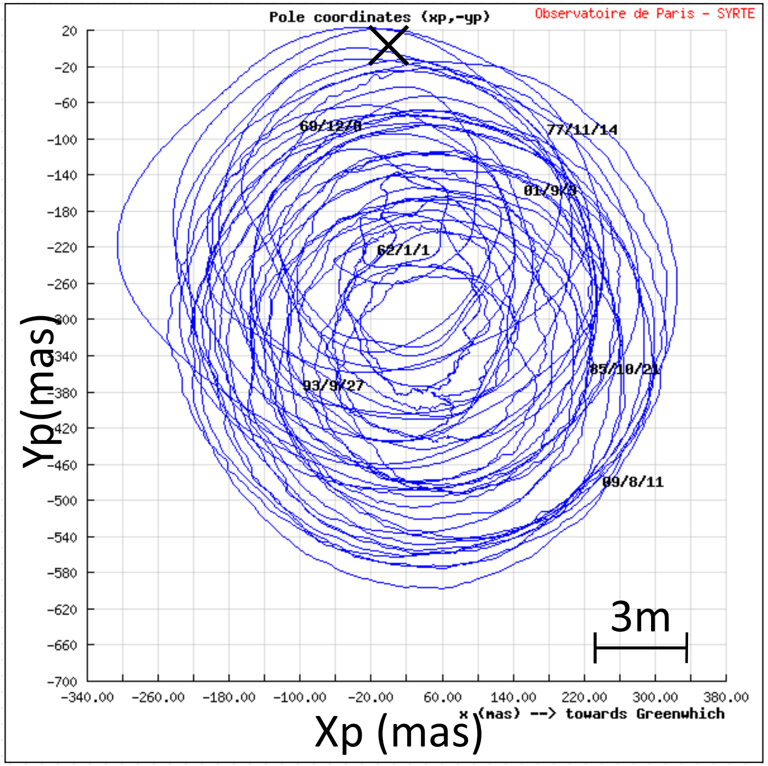
$$\boldsymbol{r}_{eci} = \boldsymbol{PNR}_z(-GST)\boldsymbol{Wr}_{ecef}$$

$$= \underbrace{\boldsymbol{R}_Z(\zeta_A)\boldsymbol{R}_Y(-\theta_A)\boldsymbol{R}_Z(z_A)}_{(3)}\underbrace{\boldsymbol{R}_X(-\varepsilon_A)\boldsymbol{R}_Z(\Delta\varphi)\boldsymbol{R}_X(\varepsilon_A+\Delta\varepsilon)}_{(2)}\underbrace{\boldsymbol{R}_Z(-GST)}_{(1)}\underbrace{\boldsymbol{R}_X(y_P)\boldsymbol{R}_Y(x_P)}\boldsymbol{r}_{ecef}$$

EOP: Earth Orientation Parameters

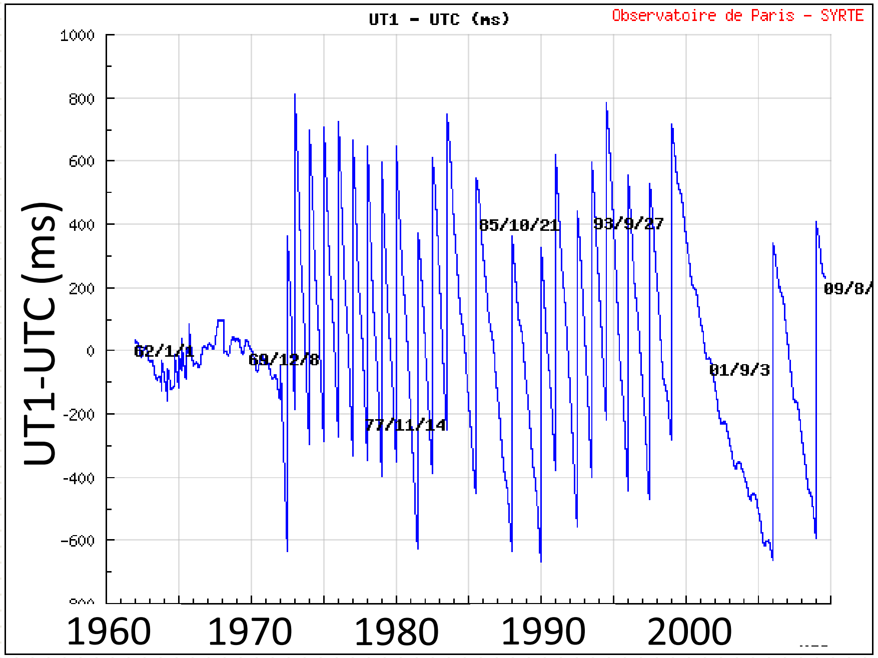
Polar Motion:

Xp, Yp



Earth Rotation Angle:

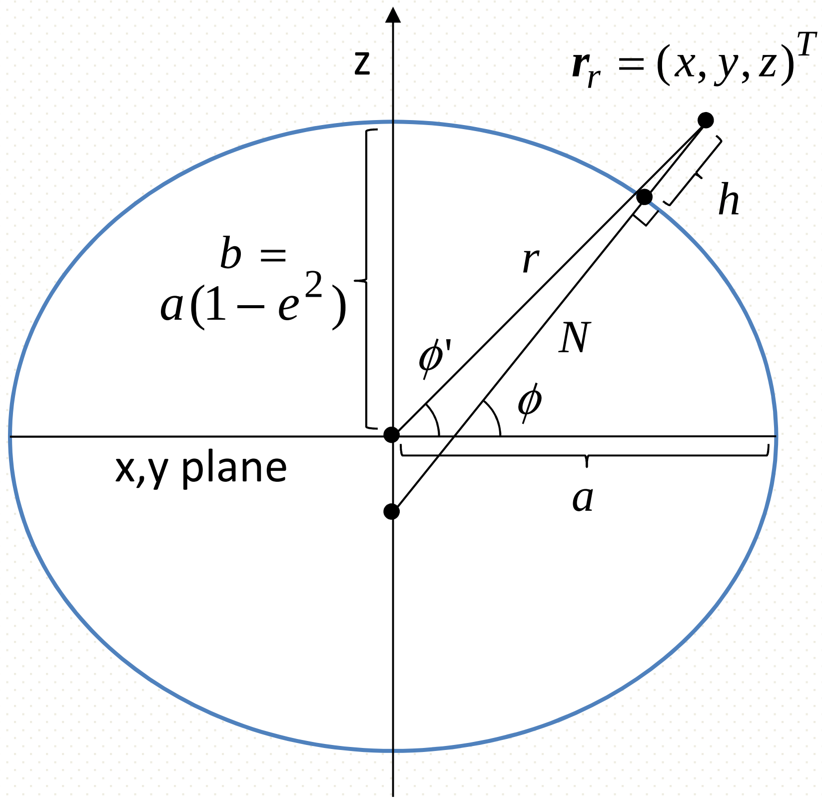
UT1-UTC



IERS C04 Series (1962/1/1-2009/8/11)

Ellipsoid and Datum

Ellipsoid:



ϕ' : Geocentric Latitude λ : Longitude

ϕ : Geodetic Latitude h : Ellipsoidal Height

	GRS 80	WGS 84
a (m)	6378137	6378137
f	1/298.257222 101	1/298.257223 563
GM (m^3/s^2)	3986005.000 x 10^8	3986004.418 x 10^8

Lat/Lon/Height to ECEF:

$$e^2 = f(2-f)$$

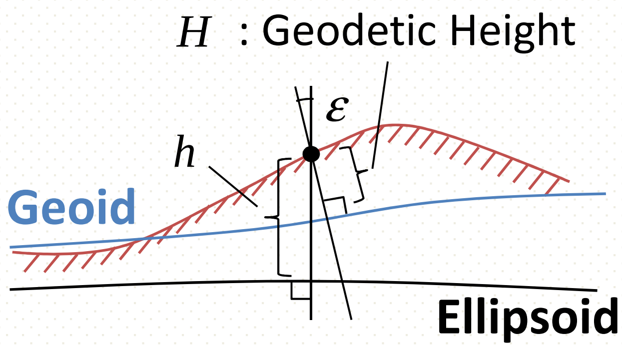
$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

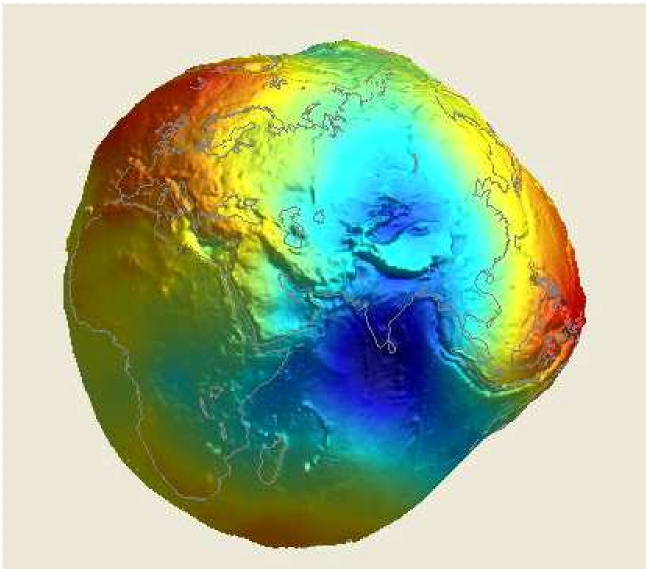
$$\boldsymbol{r}_r = \begin{pmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ (N(1+e^2)+h)\sin\phi \end{pmatrix}$$

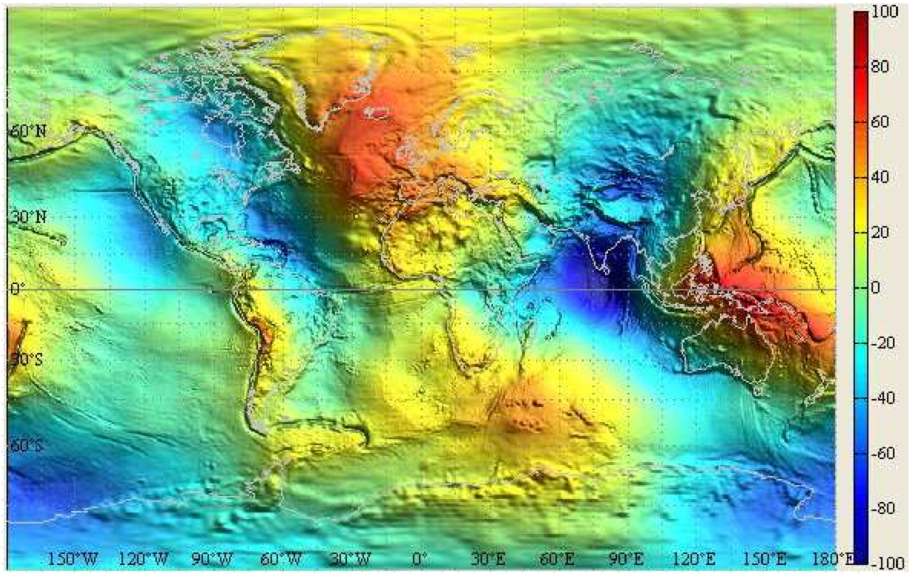
Geoid

Geopotential:

$$V(r,\phi',\lambda) = \frac{GM}{r}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n (\bar{C}_{nm}Y_{nmc} + \bar{S}_{nm}Y_{nms})\right\}$$







EGM96 Geoid Model

Spherical Harmonics

Spherical harmonic functions:

$$Y_{n0} = Y_{n0c}$$

$$Y_{nmc} = \overline{P}_{nm}(\sin\phi')\cos m\lambda$$

$$Y_{nms} = \overline{P}_{nm}(\sin\phi')\sin m\lambda$$

Legendre function:

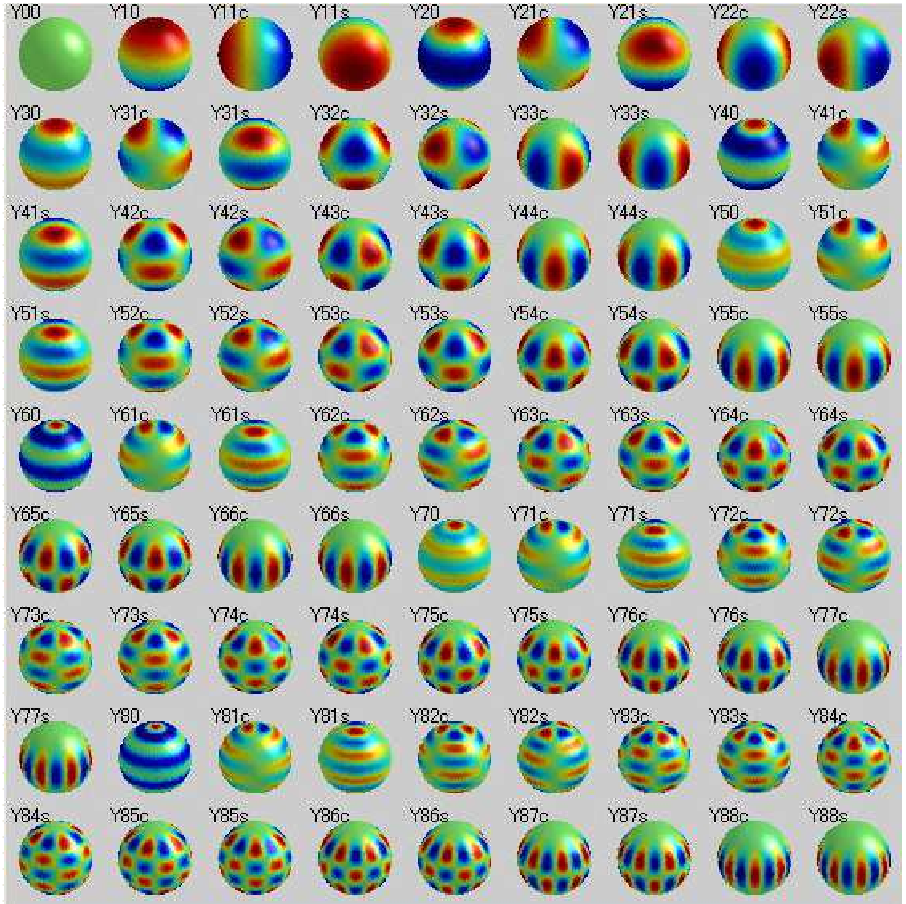
$$\overline{P}_{nm} = N_{nm}P_{nm},\ P_{00}(x) = 1, P_{10}(x) = x$$

$$P_{n-1,n}(x) = 0,$$

$$P_{nn}(x) = (2n-1)(1-x^2)^{1/2}P_{n-1,n-1}(x)$$

$$P_{nm}(x) = \frac{(2n-1)xP_{n-1,m}(x) - (n+m-1)P_{n-2,m}(x)}{n-m}$$

$$N_{nm} = \begin{cases} \sqrt{2n+1} & (m=0) \\ \sqrt{\dfrac{2(2n+1)(n-m)!}{(n+m)!}} & (m>0) \end{cases}$$



Coordinates Transformation

Helmert Transformation (A to B):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_B = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + (1+D)\begin{pmatrix} 1 & -R_3 & R_2 \\ R_3 & 1 & -R_1 \\ -R_2 & R_1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_A$$

- T1, T2, T3 : Translation along coordinate axis
- D : Scale factor
- R1, R2, R3 : Rotation of coordinate axis

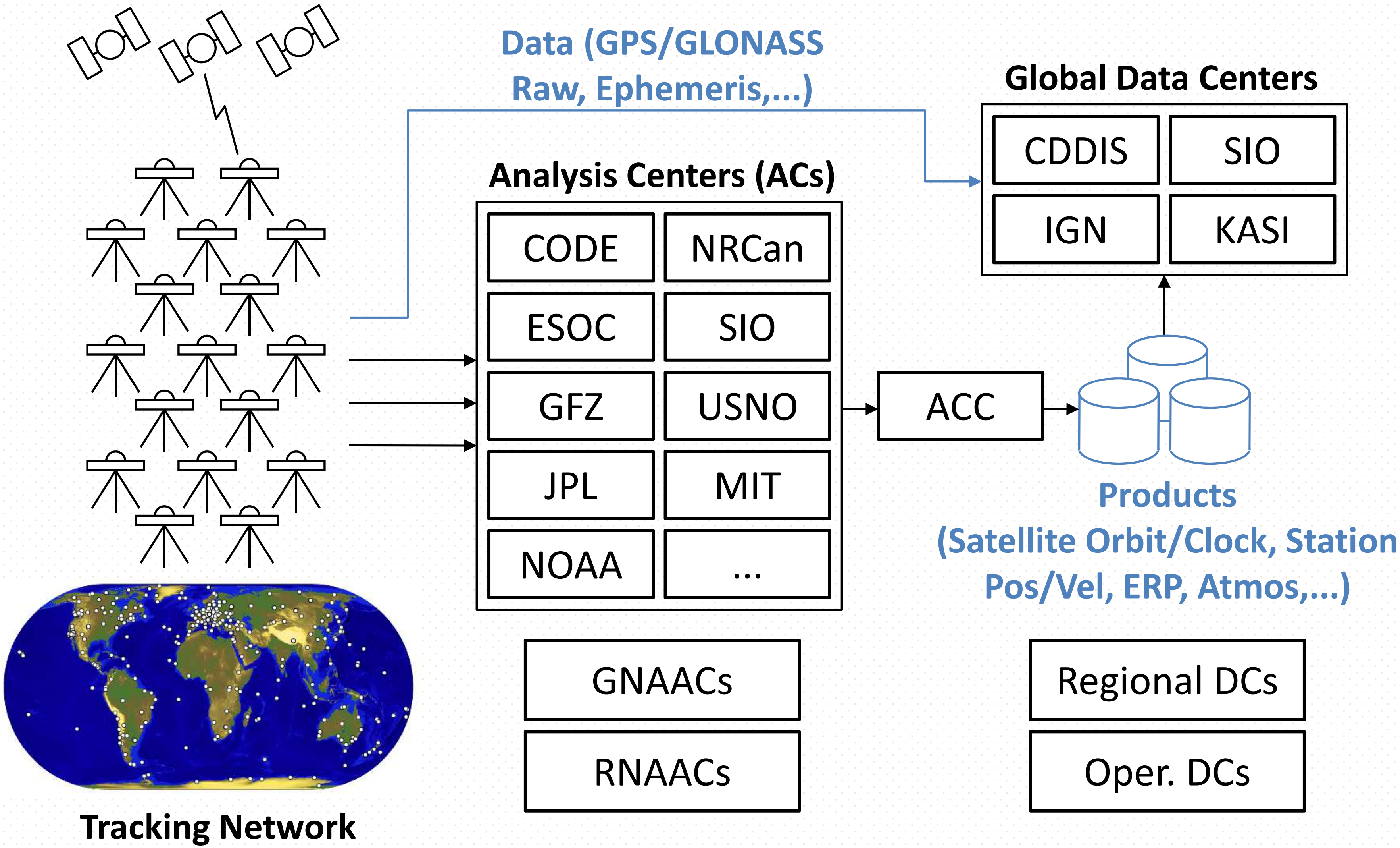
Coordinates		T1 (mm)	T2 (mm)	T3 (mm)	D (10^{-9})	R1 (mas)	R2 (mas)	R3 (mas)
A	B							
ITRF2005	ITRF2000	0.1	-0.8	-5.8	0.40	0.00	0.00	0.00
		-0.2/y	0.1/y	-1.8/y	0.08/y	0.00/y	0.00/y	0.00/y

(Epoch 2000.0)

Precise Ephemeris

- **Precise Satellite Orbit and Clock**
  - By Post-Processing or in Real-time
  - Observation Data of Tracking Stations World-Wide
- **Format:**
  - Orbit: NGS SP3
  - Clock: NGS SP3 or RINEX Clock Extension
- **Contents:**
  - Orbit: ECEF-Positions of Satellite Mass Center
  - Clock: Clock-biases wrt Time Scale Aligned to GPS Time

IGS: International GNSS Service



IGS Products

		Final (IGS)	Rapid (IGR)	Ultra-Rapid (IGU)		Broadcast
				Observed	Predicted	
Accuracy	Orbit	~2.5cm	~2.5cm	~3cm	~5cm	~100cm
	Clock	~75ps RMS ~20ps STD	~75ps RMS ~25ps STD	~150ps RMS ~50ps STD	~3ns RMS ~1.5ns STD	~5ns RMS ~2.5ns STD
Latency		12-18 days	17-41 hours	3-9 hours	realtime	realtime
Updates		every Thursday	at 17 UTC daily	at 03, 09, 15, 21 UTC	at 03, 09, 15, 21 UTC	-
Sample Interval	Orbit	15min	15min	15min	15min	daily
	Clock	Sat: 30s Stn: 5min	5min	15min	15min	daily

(2009/8, http://igscb.jpl.nasa.gov/)

Interpolation of Satellite Orbit

Lagrange Interpolation:

$$\boldsymbol{r}^s(t) = \frac{(t-t_2)(t-t_3)\ldots(t-t_{n+1})}{(t_1-t_2)(t_1-t_3)\ldots(t_1-t_{n+1})}\boldsymbol{r}^s(t_1) + \frac{(t-t_1)(t-t_3)\ldots(t-t_{n+1})}{(t_2-t_1)(t_2-t_3)\ldots(t_2-t_{n+1})}\boldsymbol{r}^s(t_2)$$
$$+ \frac{(t-t_1)(t-t_2)\ldots(t-t_{n+1})}{(t_3-t_1)(t_3-t_2)\ldots(t_3-t_{n+1})}\boldsymbol{r}^s(t_3) + \ldots + \frac{(t-t_1)(t-t_2)\ldots(t-t_n)}{(t_{n+1}-t_1)(t_{n+1}-t_2)\ldots(t_{n+1}-t_n)}\boldsymbol{r}^s(t_{n+1})$$

Interpolation Error of 15-min Sample Orbit

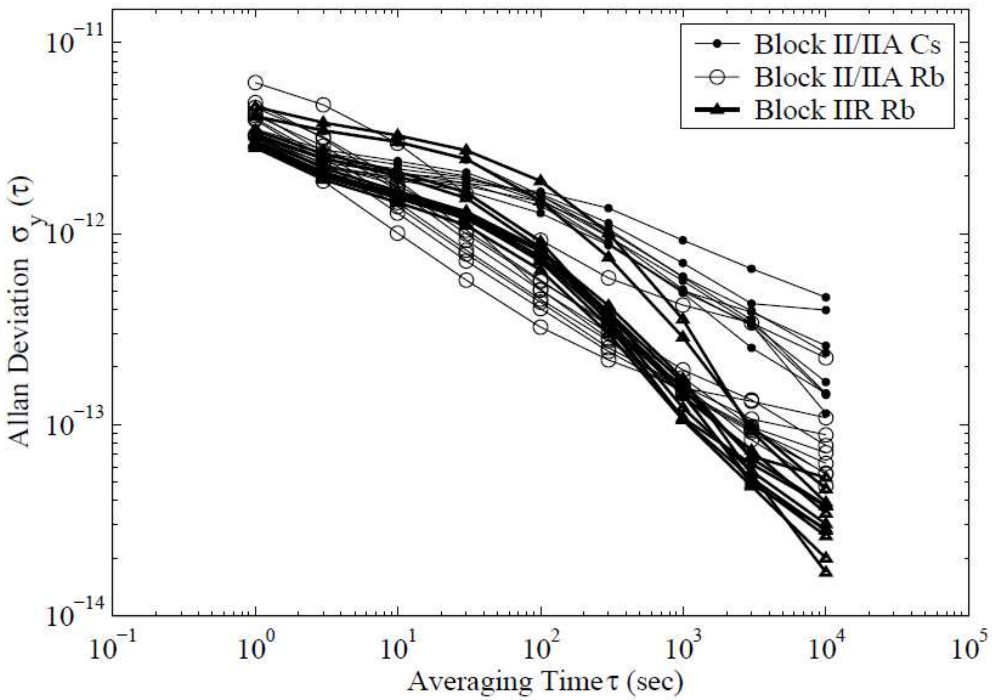
Degree of Polynomial	Position RMS Error (cm)			Velocity RMS Error (cm/s)		
	Radial	Along-Trk	Cross-Trk	Radial	Along-Trk	Cross-Trk
n=5	72.10	73.84	57.48	0.253	0.260	0.202
n=6	7.31	6.89	5.75	0.032	0.031	0.025
n=7	0.63	0.63	0.50	0.017	0.019	0.014
n=8	0.08	0.11	0.08	0.017	0.018	0.013
n=9	0.05	0.11	0.05	0.017	0.018	0.013
n=10	**0.05**	**0.10**	**0.06**	**0.017**	**0.018**	**0.013**
n=11	0.05	0.12	0.06	0.017	0.018	0.013

Interpolation of Satellite Clock

Linear Interpolation:

$$dT^S(t) = \frac{(t_2 - t)\,dT^S(t_1) + (t - t_1)\,dT^S(t_2)}{t_2 - t_1} + \Delta t_{rel}$$

Satellite Clock Stability:



Ionospheric Delay

Ionospheric Delay Model:

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{{Y_T}^2}{2(1-X-iZ)} \pm \sqrt{\frac{{Y_T}^4}{4(1-X-iZ)^2} + {Y_L}^2}} \approx 1 - X = 1 - f_N^2 / f^2 \ (L-band)$$

: Appleton-Hartree Formula

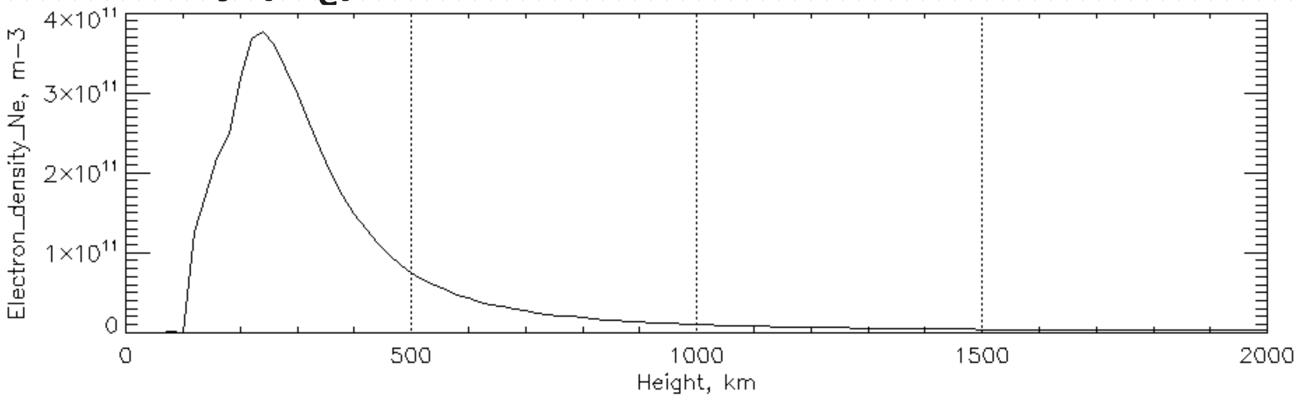
$$n = \sqrt{1 - f_N^2 / f^2} \approx 1 - f_N^2 / 2f^2 = 1 - 40.30 N_e / f^2$$

$$f_N^2 = \frac{N_e e^2}{4\pi^2 \varepsilon_0 m_e}$$: plasma frequency

$$I_r^s \approx \int 40.30 N_e / f^2 dl = 40.30 \times 10^{16} TEC / f^2$$

TEC: Total Electron Content

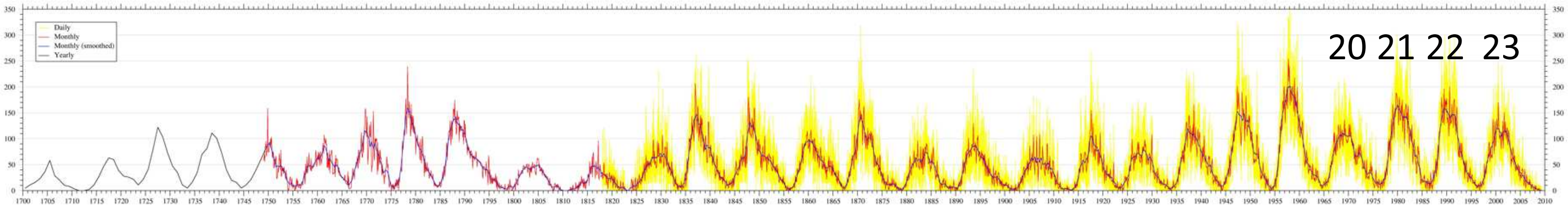
Electron Density (N_e):



IRI-2007 model: 2009/7/31 0:00 Tokyo (http://modelweb.gsfc.nasa.gov/models/iri.html)

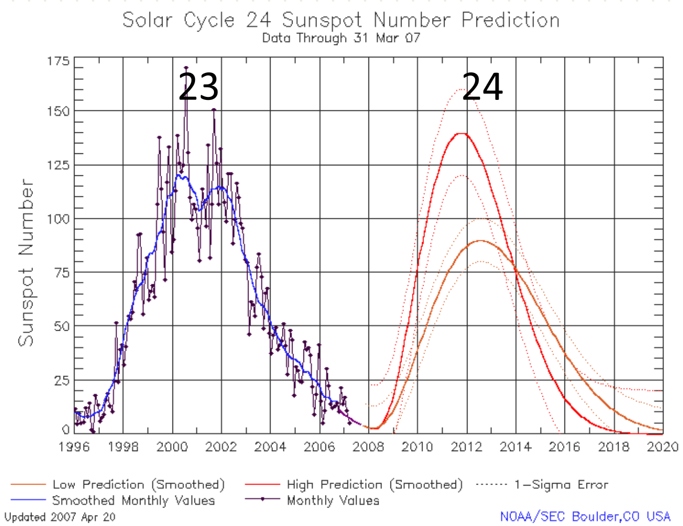
Solar Cycle

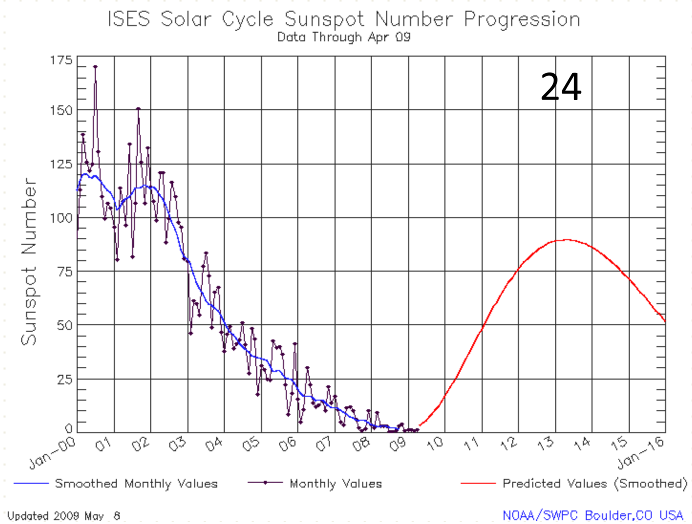
International Sunspot Number (ISN): 1700-2009



by SIDC (Solar Influences Data Analysis Center) in Belglum (http://sidc.oma.be)

Solar Cycle Prediction: Cycle 24





by NOAA SWPC (Space Weather Prediction Center) (http://www.swpc.noaa.gov/SolarCycle)

LC: Linear Combination

$$C = a\Phi_1 + b\Phi_2 + cP_1 + dP_2 (\Phi_1 = \lambda_1\phi_1,\ \Phi_2 = \lambda_2\phi_2)$$

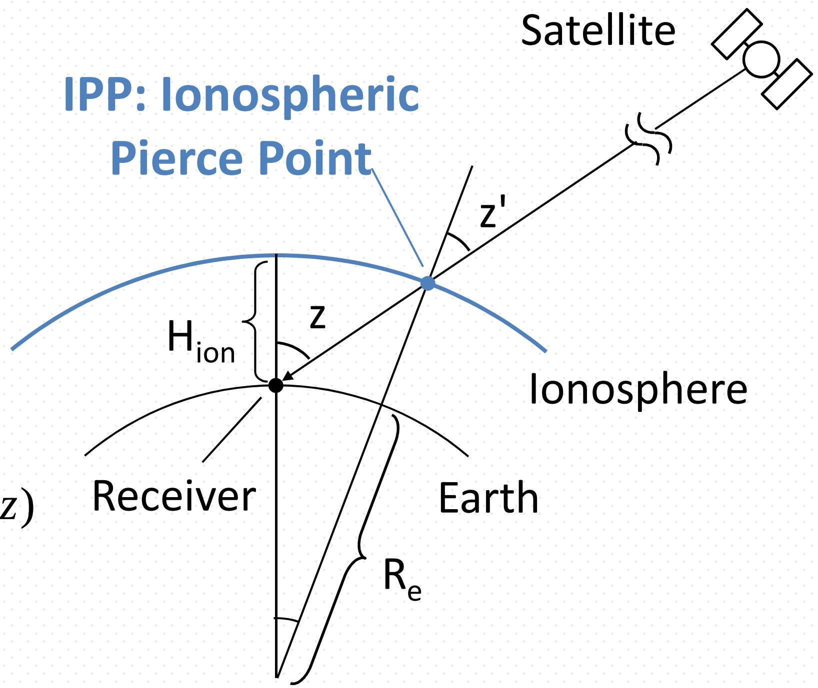
	LC	Coefficients				Wave Length (cm)	Ionos Effect wrt L1	Typical Noise (cm)
		a	*b*	*c*	*d*			
L1	L1 Carrier-Phase	1	0	0	0	19.0	1.0	0.3
L2	L2 Carrier-Phase	0	1	0	0	24.4	1.6	0.3
LC/L3	**Iono-Free Phase**	C_1	C_2	0	0	-	**0.0**	**0.9**
LG/L4	Geometry-Free Phase	1	-1	0	0	-	0.6	0.4
WL	Wide-Lane Phase	λ_W/λ_1	$-\lambda_W/\lambda_2$	0	0	86.2	1.3	1.7
NL	Narrow-Lane Phase	λ_N/λ_1	λ_N/λ_2	0	0	10.7	1.3	1.7
MW	Melbourne-Wübbena	λ_W/λ_1	$-\lambda_W/\lambda_2$	λ_N/λ_1	λ_N/λ_2	86.2	0.0	21
MP1	L1-Multipath	$2C_2-1$	$-2C_2$	1	0	-	0.0	30
MP2	L2-Multipath	$-2C_1$	$2C_1-1$	0	1	-	0.0	30

$$C_1 = f_1^2/(f_1^2 - f_2^2),\ C_2 = -f_2^2/(f_1^2 - f_2^2),\ \lambda_W = 1/(1/\lambda_1 - 1/\lambda_2),\ \lambda_N = 1/(1/\lambda_1 + 1/\lambda_2)$$

Single Layer Model

Ionospheric Delay Model:

$$I = \frac{40.30\times10^{16}}{f^2}TEC \approx \frac{1}{\cos z'}\frac{40.30\times10^{16}}{f^2}\times VTEC(t,\phi_{pp},\lambda_{pp})$$



IPP Position/Slant Factor:

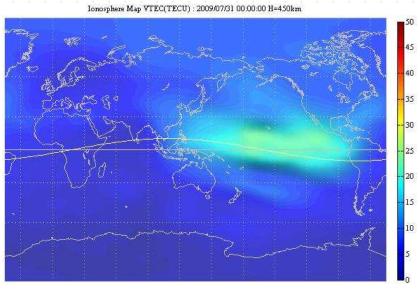
$$z = \pi/2 - El$$

$$z' = \arcsin\frac{R_e \sin z}{R_e + H_{ion}}, \alpha = z - z'$$

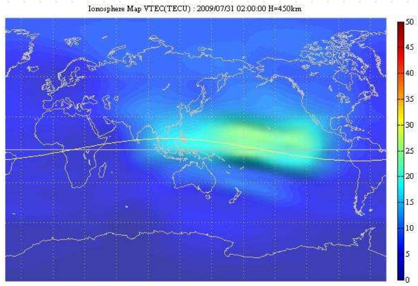
$$\phi_{pp} = \arcsin(\cos\alpha\sin\phi + \sin\alpha\cos\phi\cos Az)$$

$$\lambda_{pp} = \lambda + \arcsin\frac{\sin\alpha\sin Az}{\phi_{pp}}$$

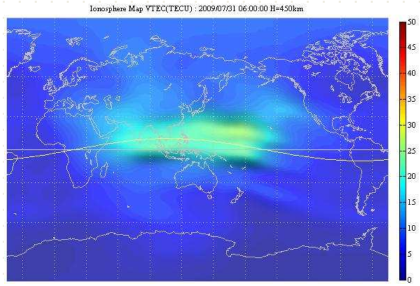
Ionospheric TEC Grid



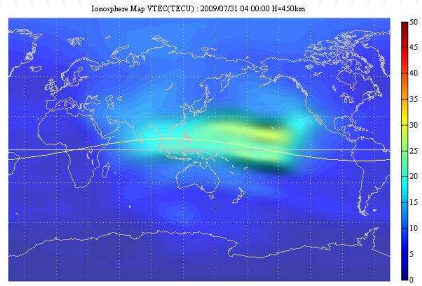
2009/7/31 0:00



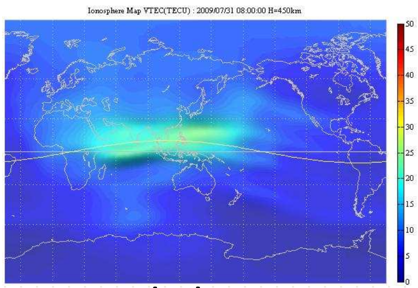
2009/7/31 2:00



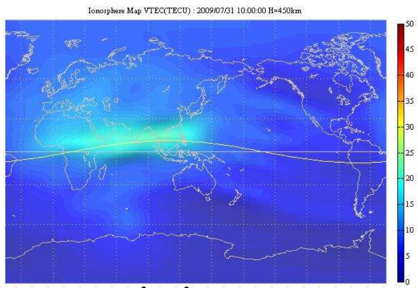
2009/7/31 4:00



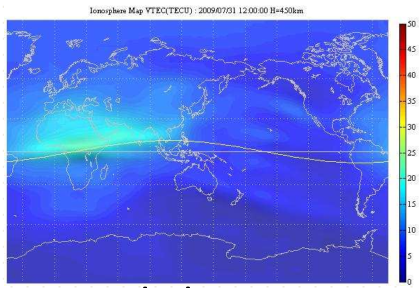
2009/7/31 6:00



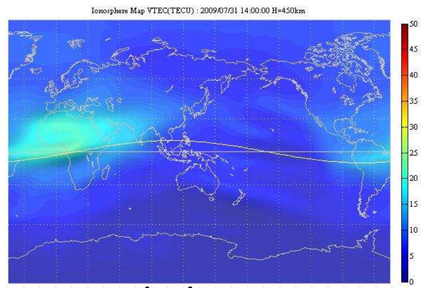
2009/7/31 8:00



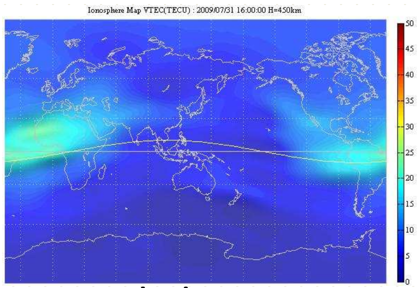
2009/7/31 10:00



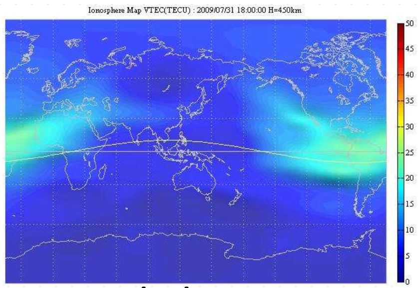
2009/7/31 12:00



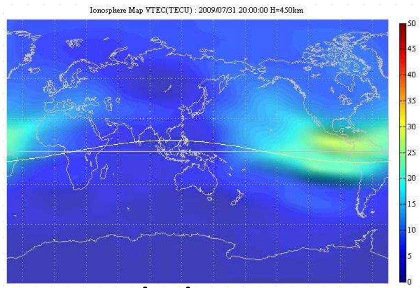
2009/7/31 14:00



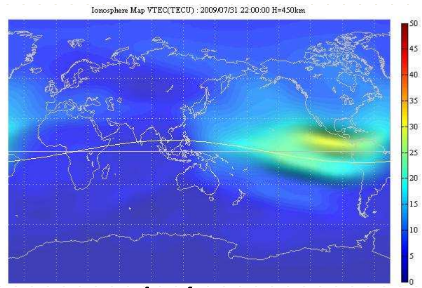
2009/7/31 16:00



2009/7/31 18:00



2009/7/31 20:00



2009/7/31 22:00

(IGS TEC Final, GPS Time)

Tropospheric Delay

Tropospheric Delay Model:

$$T = m_h(El)ZHD + m_w(El)ZWD$$

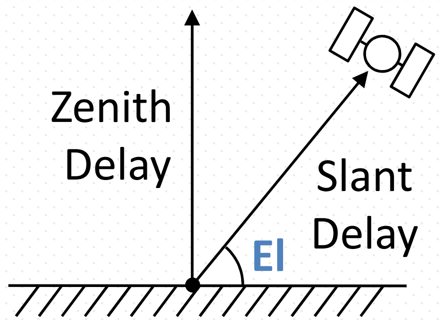
$$ZHD = \frac{0.0022768\,p}{1 - 0.00266\cos 2\phi - 2.8\times 10^{-7} H}$$

: Zenith Hydrostatic Delay (m)

ZWD : Zenith Wet Delay (m)

$m_h(El)$: Hydrostatic Mapping Function

$m_w(El)$: Wet Mapping Function



ZWD to PWV (Precipitable Water Vapor):

$$T_m = 70.2 + 0.72T$$

$$PWV = \frac{1\times 10^5}{R_v\left(k_2 - k_1\frac{m_v}{m_d} + \frac{k_3}{T_m}\right)} ZWD$$

$R_v = 461, k_1 = 77.6,$

$k_2 = 71.98, k_3 = 3.754\times 10^5$

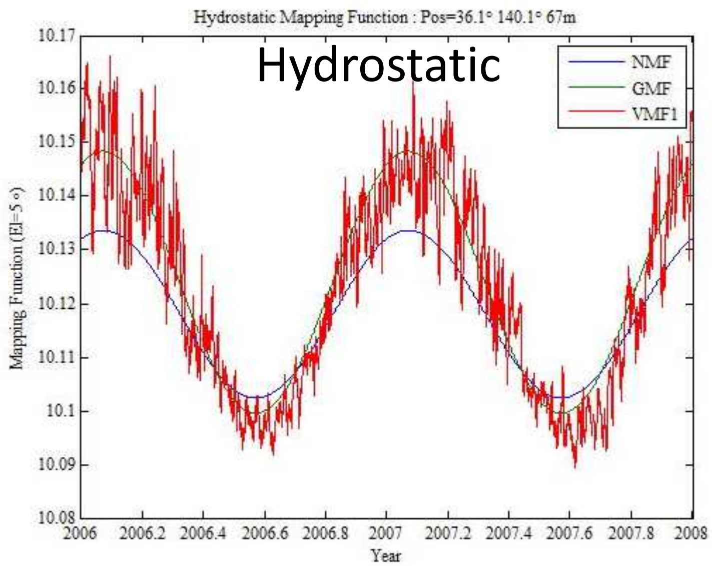
$m_v = 18.0152, m_d = 28.9644$

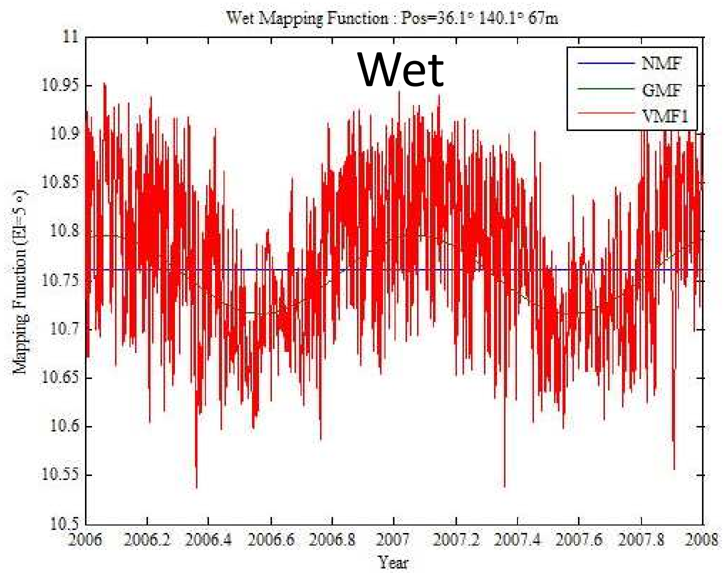
Mapping Function

$$m(El) = \frac{1 + \cfrac{a}{1 + \cfrac{b}{1 + c}}}{\sin(El) + \cfrac{a}{\sin(El) + \cfrac{b}{\sin(El) + c}}}$$

a, b, c : Mapping Function Coefficients

NMF, GMF, VMF1





(2006/1/1-2007/12/31, TSKB, El=5deg)

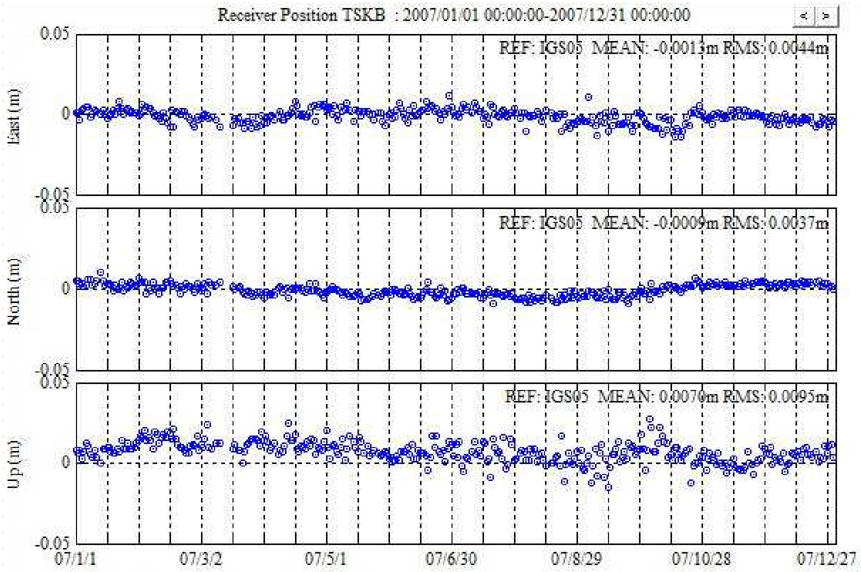
Tropospheric Gradient

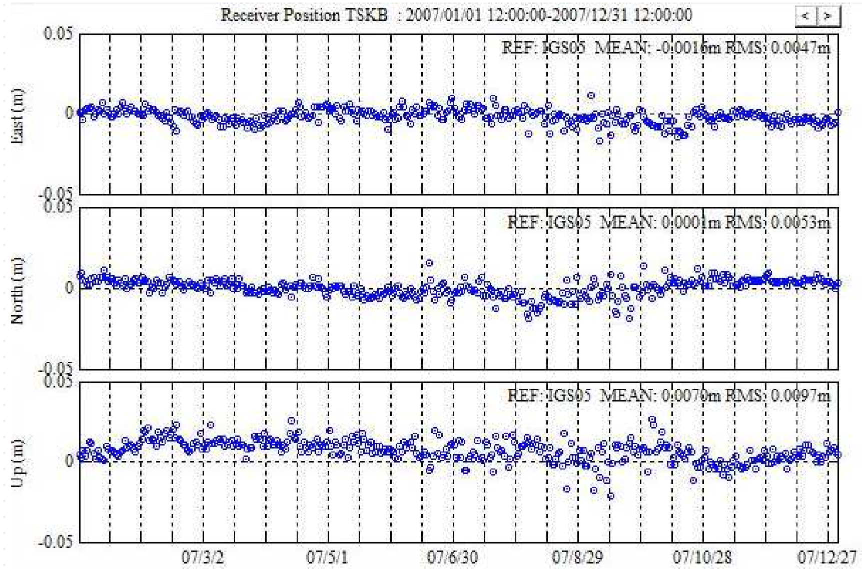
Mapping Function with Horizontal Gradient:

$$m(El, Az) = m_0(El) + m_0(El)\cot(El)\left(G_N \cos(Az) + G_E \sin(Az)\right)$$

G_N, G_E : North/East Gradient Parameters

PPP Solutions with Gradient Estimation	**PPP Solutions without Gradient Estimation**

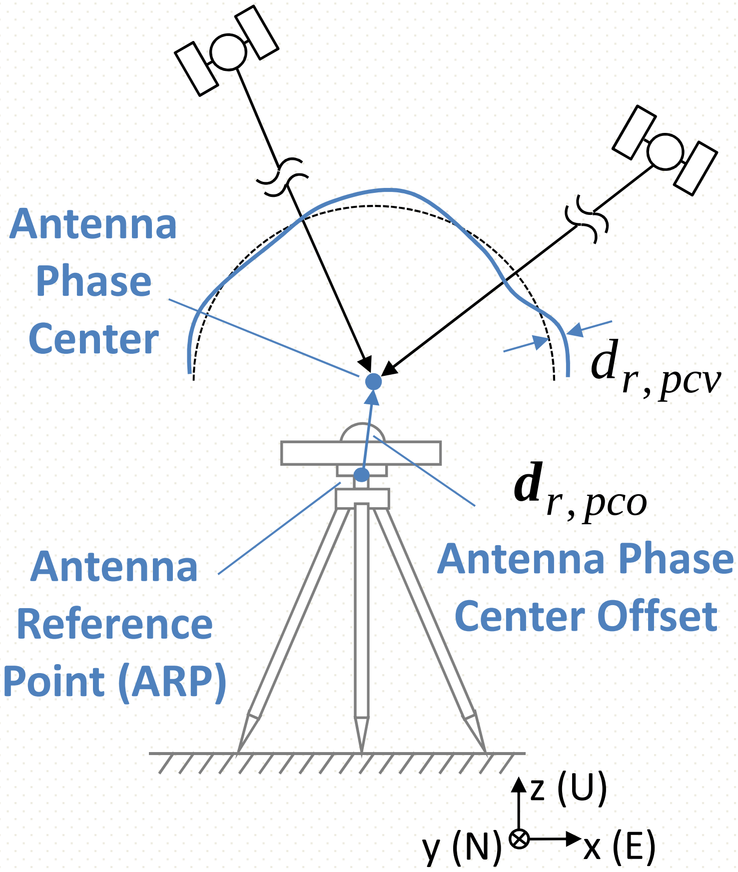




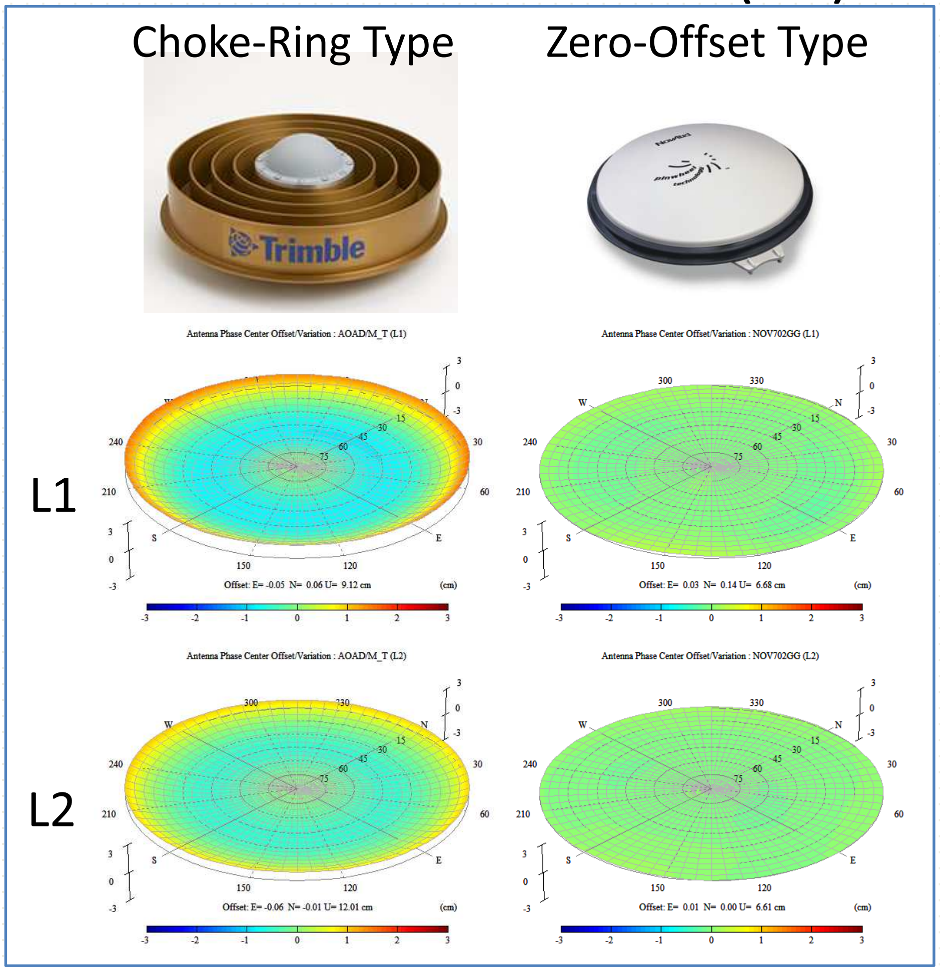
2007/1/1-12/31, 24H-Static PPP, TSKB

Antenna Phase Center 1

Receiver Antenna Phase Center:



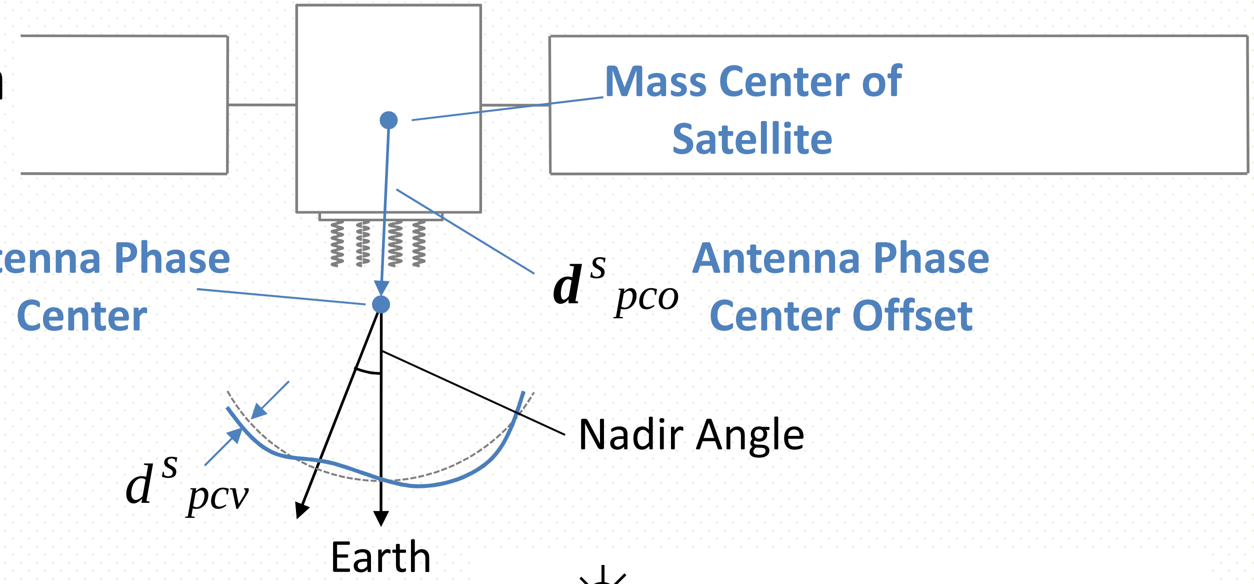
Antenna Phase Center Variation (PCV)



IGS Absolute Antenna Model (IGS05.PCV)

Antenna Phase Center 2

Satellite Antenna Phase Center:

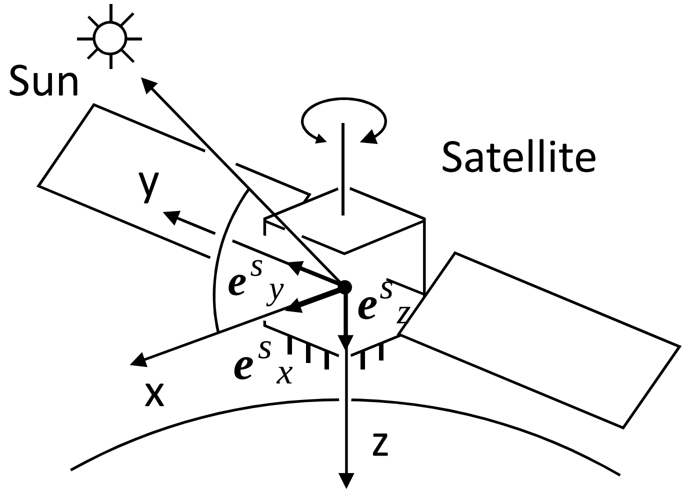


Satellite Coordinate to ECEF:

$$E_{sat \to ecef} = (e^s{}_x, e^s{}_y, e^s{}_z)$$

$$e^s{}_z = -\frac{r^s}{\left\| r^s \right\|}, e_s = \frac{r_{sun} - r^s}{\left\| r_{sun} - r^s \right\|}$$

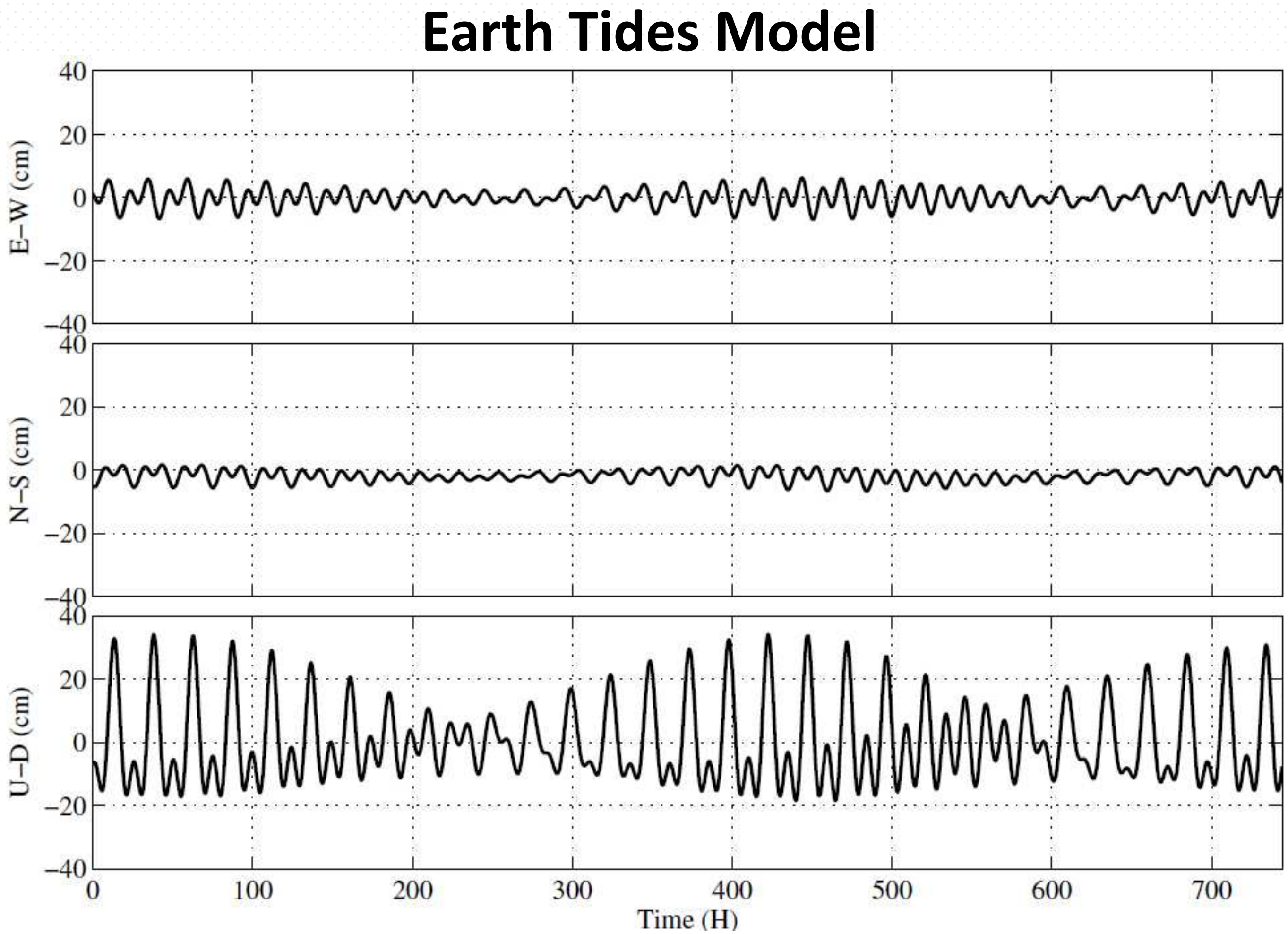
$$e^s{}_y = \frac{e^s{}_z \times e^s{}_s}{\left\| e^s{}_z \times e_s \right\|}, e^s{}_x = e^s{}_y \times e^s{}_z$$



Site Displacement

- **Displacement of Ground-Fixed Receiver**
  - Solid Earth Tide
  - Ocean Tide Loading (OTL)
  - Pole Tide
  - Atmospheric Loading
- **Tide Model**
  - IERS Conventions 1996/2003/2010
  - Ocean Loading: Schwiderski, GOT99.2/00.2, CSR 3.0/4.0, FES99/2004, NAO99.b
  - M_2,S_2,N_2,K_2,K_1,O_1,P_1,Q_1,M_1,M_m,S_{sa}

Earth Tides



IERS Conventions 1996 + NAO99.b, 2007/1/1-1/31, TSKB

Phase Wind-up Effect

- Relative rotation between satellite and receiver antennas effect to the measured phase of RHCP signal.

$$d_{pw} = \lambda \left\{ sign(\boldsymbol{e}_r^s \cdot (\boldsymbol{D}^s \times \boldsymbol{D}_r)) \arccos \frac{\boldsymbol{D}^s \cdot \boldsymbol{D}_r}{\left\| \boldsymbol{D}^s \right\| \left\| \boldsymbol{D}_r \right\|} / 2\pi + N \right\}$$

$\boldsymbol{D}^s = \boldsymbol{e}_x^s - \boldsymbol{e}_u^s(\boldsymbol{e}_u^s \cdot \boldsymbol{e}_x^s) - \boldsymbol{e}_u^s \times \boldsymbol{e}_y^s$: Dipole Vector of Satellite Antenna

$\boldsymbol{D}_r = \boldsymbol{e}_{r,x} - \boldsymbol{e}_r^s(\boldsymbol{e}_r^s \cdot \boldsymbol{e}_{r,x}) + \boldsymbol{e}_r^s \times \boldsymbol{e}_{r,y}$: Dipole Vector of Receiver Antenna

$\boldsymbol{E}_{ecef \to enu} = (\boldsymbol{e}_{r,x}{}^T, \boldsymbol{e}_{r,y}{}^T, \boldsymbol{e}_{r,z}{}^T)^T$: ECEF to ENU Transformation Matrix

$\boldsymbol{e}_r^s$: LOS Vector from Receiver to Satellite Antenna

N : Integer Ambiguity

Relativistic Effects

- **Satellite/Receiver:**
  - Frequency Shift by Earth Gravity (General Rel.)
  - Frequency Shift by Sun/Moon Gravity (General Rel.)
  - Second-Order Doppler-Shift by Motion (Special Rel.)
- **Signal Propagation:**
  - Sagnac Correction (Rotating Coordinates)
  - Shapiro Time Delay Effect
  - Lense-Thirring Drag

Satellite Clock Bias/Rate Correction
+ Periodic Term:

$$d_{rel} = -\frac{2\boldsymbol{r}^s \cdot \boldsymbol{v}^s}{c^2}$$

Standard Data Format

- **RINEX (Receiver Independent Echange)**
  - Text-based Standard GNSS data file format
  - Mainly for post-processing
- **RINEX Types**
  - OBS: Observation data
  - NAV: navigation data, (GNAV: GLONASS, HNAV: SBAS)
  - MET: Meteorological data
  - CLK: Clock product
- **RINEX Version**
  - ver. 2 (2.10, 2.11, 2.12), ver. 3 (3.00, 3.01, 3.02m, ...)

RINEX OBS (Observation Data)

```
     2.10           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
RTKCONV 2.4.0                           20110423 090647 UTC PGM / RUN BY / DATE
                                                            MARKER NAME
                                                            MARKER NUMBER
                                                            OBSERVER / AGENCY
                                                            REC # / TYPE / VERS
                                                            ANT # / TYPE
        0.0000        0.0000        0.0000                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     8    C1    L1    D1    S1    P2    L2    D2    S2      # / TYPES OF OBSERV
  2010    10    15     0     0    0.0000000     GPS         TIME OF FIRST OBS
  2010    10    15     2    28   54.0000000     GPS         TIME OF LAST OBS
                                                            END OF HEADER
 10 10 15  0  0  0.0000000  0 10G 6G23G16G19G21G13G 3G31S29S37
  20849928.484  109567124.316       1939.684        45.000    20849930.125
  85377001.480       1511.441         41.000
  22450960.859  117980618.953       1062.035        42.000    22450959.898
  91932917.910        827.555         38.000
  20790247.117  109253470.496        334.336        45.000    20790246.844
  85132587.789        260.520         41.000
  24794846.031  130297776.969       3763.289        38.000    24794848.422
 101530723.414       2932.430         32.000
  23378478.469  122854746.020        860.133        40.000    23378477.977
  95730986.191        670.234         34.000
  24155219.492  126936537.238       2611.234        35.000    24155223.109
  98911564.082       2034.727         33.000
  21765068.656  114376223.133       3035.375        42.000    21765071.242
  89124339.934       2365.223         38.000
  21044041.703  110587188.461      -1456.918        45.000    21044041.797
  86171830.961      -1135.266         42.000
  37172827.633  195344531.559          2.965        38.000

  37203973.328  195508183.188         -0.992        39.000

 10 10 15  0  0  1.0000000  0 10G 6G23G16G19G21G13G 3G31S29S37
  20849559.430  109565184.891       1939.090        45.000    20849561.062
```

Receiver Time Tag

Types of OBS
C,P*: Pseudorange
L*: Carrier-phase
D*: Doppler Freq
S*: CN0 (dBHz)

Satellite List
nn, Gnn: GPS
Rnn: GLONASS
Jnn: QZSS
Enn: Galileo
Snn: SBAS

RINEX NAV (Navigation Data)

```
  2.10           N: GPS NAV DATA                             RINEX VERSION / TYPE
 RTKCONV 2.4.0                           20110423 090647 UTC PGM / RUN BY / DATE
     1.1176E-08  0.0000E+00 -5.9605E-08  0.0000E+00          ION ALPHA
     9.0112E+04  0.0000E+00 -1.9661E+05  0.0000E+00          ION BETA
     -.838190317154E-08 -.310862446895E-13    61440     1606 DELTA-UTC: A0,A1,T,W
     15                                                      LEAP SECONDS
                                                             END OF HEADER
 31 10 10 15  2  0  0.0 -.724568963051E-06  .352429196937E-11  .000000000000E+00
      .810000000000E+02  .105937500000E+02  .427089218552E-08 -.148856857180E+01
      .571832060814E-06  .746127020102E-02  .472925603390E-05  .515378055573E+04
      .439200000000E+06 -.176951289177E-06  .679765366385E-02  .540167093277E-07
      .978380240916E+00  .300062500000E+03 -.105249752834E+01 -.819426989566E-08
      .142863093678E-10  .100000000000E+01  .160500000000E+04  .000000000000E+00
      .240000000000E+01  .000000000000E+00 -.130385160446E-07  .810000000000E+02
      .432006000000E+06  .000000000000E+00
  6 10 10 15  2  0  0.0  .455596484244E-03 -.140971678775E-10  .000000000000E+00
      .230000000000E+02 -.352500000000E+02  .500699427569E-08  .227090783348E+01
     -.185333192348E-05  .616293260828E-02  .853091478348E-05  .515365624428E+04
      .439200000000E+06  .104308128357E-06  .204411629865E+01  .353902578354E-07
      .934819176502E+00  .200625000000E+03 -.936257940341E+00 -.811783814054E-08
      .169649923743E-09  .100000000000E+01  .160500000000E+04  .000000000000E+00
      .240000000000E+01  .000000000000E+00 -.512227416039E-08  .230000000000E+02
      .432006000000E+06  .000000000000E+00
 ...

         PRN Toc             SV_clock_bias    SV_clock_drift   SV_clock_drift_rate
             IODE            Crs              Delta_n          M0
             Cuc             e                Cus              sqrt(A)
             Toe             Cic              OMEGA            Cis
             i0              Crc              omega            OMEGA_DOT
             IDOT            Codes_on_L2_ch   GPS_Week_#       L2_P_data_flag
             SV_accuracy     SV_health        TGD              IODC
             Trans_Time      Fit_interval     spare            spare
```

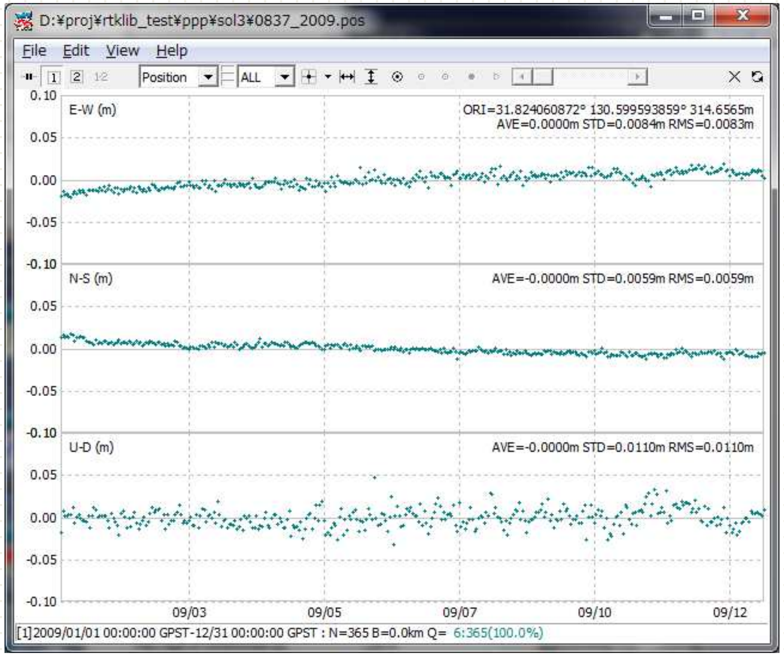
PPP (Precise Point Positioning)

- **Feature**
  - with Single Receiver (No Reference Station)
  - Efficient Analysis for Many Receivers
  - Precise Ephemeris
  - Conventionally Post-Processing
- **Applications**
  - GPS Seismometer
  - GPS Meteorology
  - POD (Precise Orbit Determination) of LEO Satellite
  - Precise Time Transfer

Static PPP vs Kinematic PPP

Static PPP Results

Station: GEONET 0837

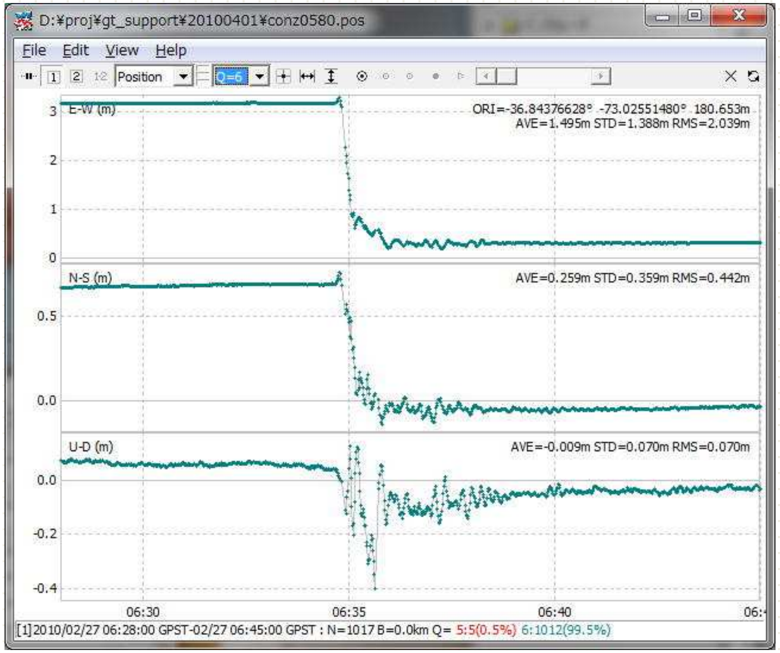


2009/1/1-2009/12/31

Interval: 1day

Kinematic PPP Results

Station: IGS CONZ



2010/2/27 6:28-6:45 GPST

Interval: 1 s

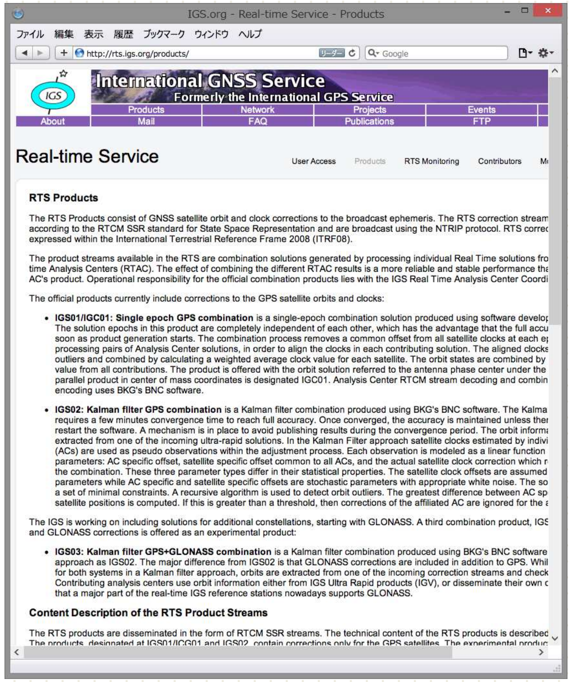
Real-time PPP

Commercial RT-PPP/GDPS Services

Service	Provider	Communication		Ref. Stations	Orbit/ Clock	Engine	Accuracy
		Coverage	Link				
StarFire	NavCom	World-wide	3 GEO L-band	60	1 min/ 1-2 s	JPL RTG	<10 cm H <15 cm V (1 sigma)
OmniSTAR XP/HP+	Fugro	World-wide (Land)	6 GEO L-band	100	1 min/ 10 s	Fugro	dm-class
SeaSTAR XP/G2	Fugro	World-wide (Sea)	6 GEO L-band	100	1 min/ 10 s	Fuguro/ ESOC (G2)	dm-class
VERIPOS Ultra/Apex	VERIPOS	World-wide	7 GEO L-band	80	30 s/ 30 s	JPL/ ESOC	10 cm H 20 cm V (95%)

IGS Real-time Service

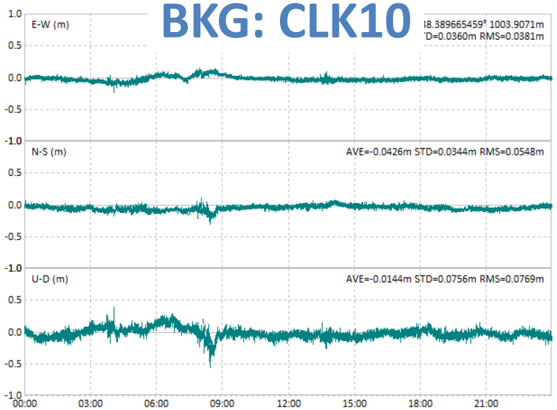
- **Developed by IGS-RTPP**
  - RTCM v.3 MT1057-1068 (SSR)
  - Corrections to broadcast ephemeris
  - Real-time NTRIP stream
  - Interval: 10 s, Latency: 5 - 10 s
  - GPS and GLONASS
- **Analysis Strategy**
  - Orbit: fixed to IGU or estimated
  - Clock: estimated with IGS real-time tracking network



http://rts.igs.org

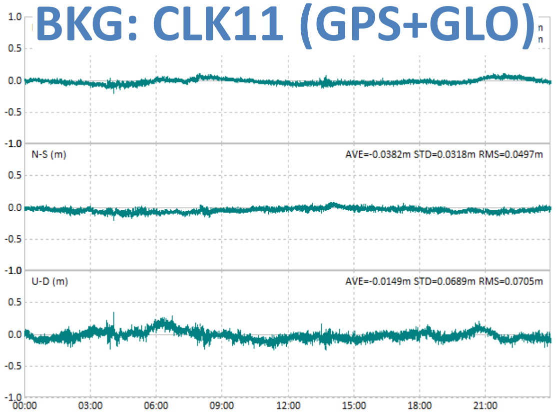
RT-PPP Performance with IGS

BKG: CLK10



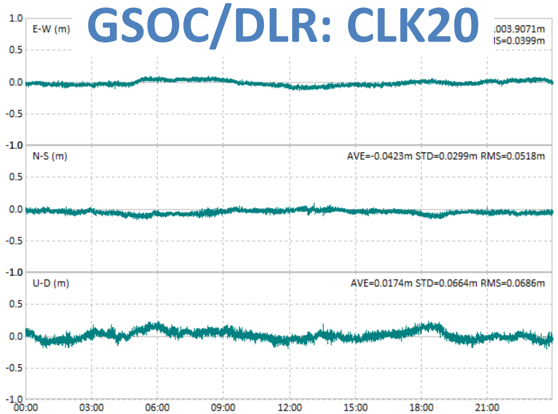
RMS: 3.8, 5.5, 7.7cm

BKG: CLK11 (GPS+GLO)



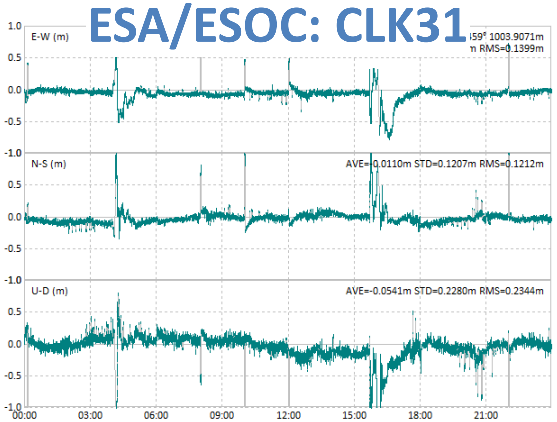
RMS: 3.8, 5.0, 7.1cm

GSOC/DLR: CLK20



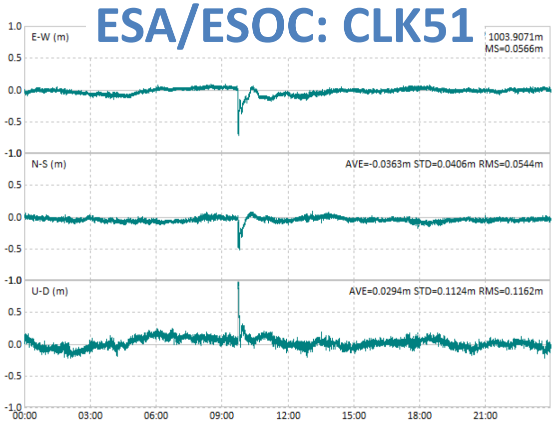
RMS: 4.0, 5.2, 6.9cm

ESA/ESOC: CLK31



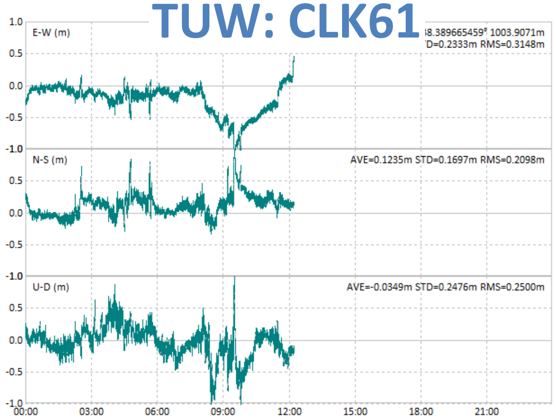
RMS: 14.0, 12.1, 23.4cm

ESA/ESOC: CLK51



RMS: 5.7, 5.4, 11.6cm

TUW: CLK61



RMS: 23.3, 21.0, 25.0cm

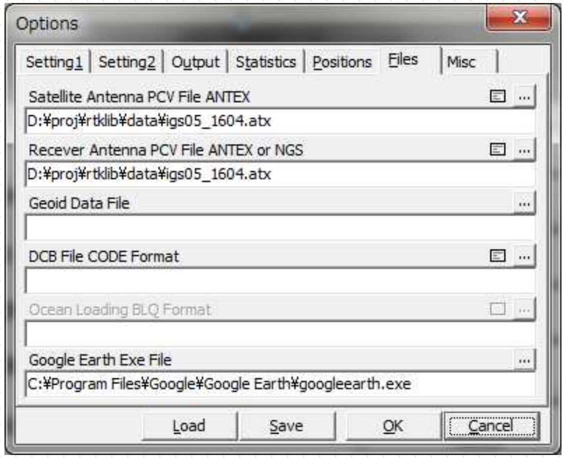
2010/9/18 0:00-23:59, 1Hz, Kinematic PPP, NovAtel OEMV-3+GPS-702, RTKLIB 2.4.1

B-7
Exercise of PPP

PPP Analysis for Reference Point

- **Objective**
  PPP Analysis for Reference Point
- **Program**
  rtklib_2.4.2¥bin¥rtkpost.exe
- **Data**
  sample4¥
  09160700.11o, 09160700.11n (RINEX)
  21100700.11o, 21100700.11n (RINEX)
  igs16265.sp3 (Precise Orbit)
  igs16265.clk_30s (Precise Clock)

RTKPOST

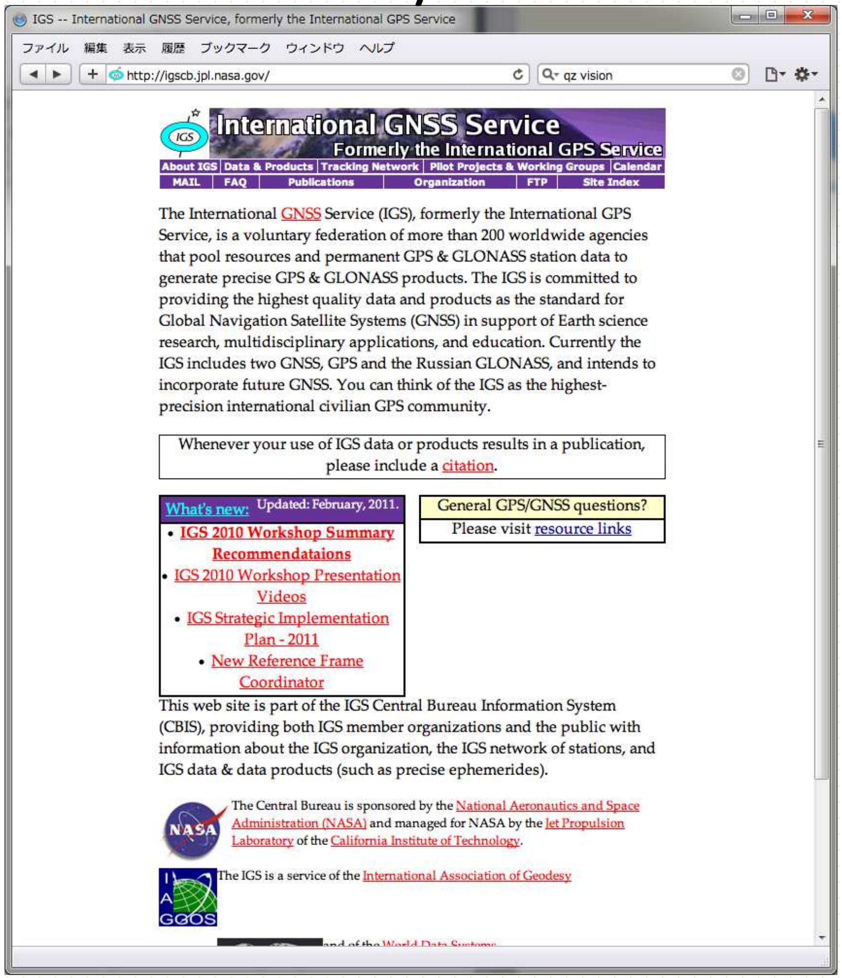


Date:
2011/3/11 GPST
(GPS week 1626-5, DOY 070)
GEONET Station:
020916: Minamikata
92110: Tsukuba

Acknowledgment: Sample Data are provided by GSI and IGS

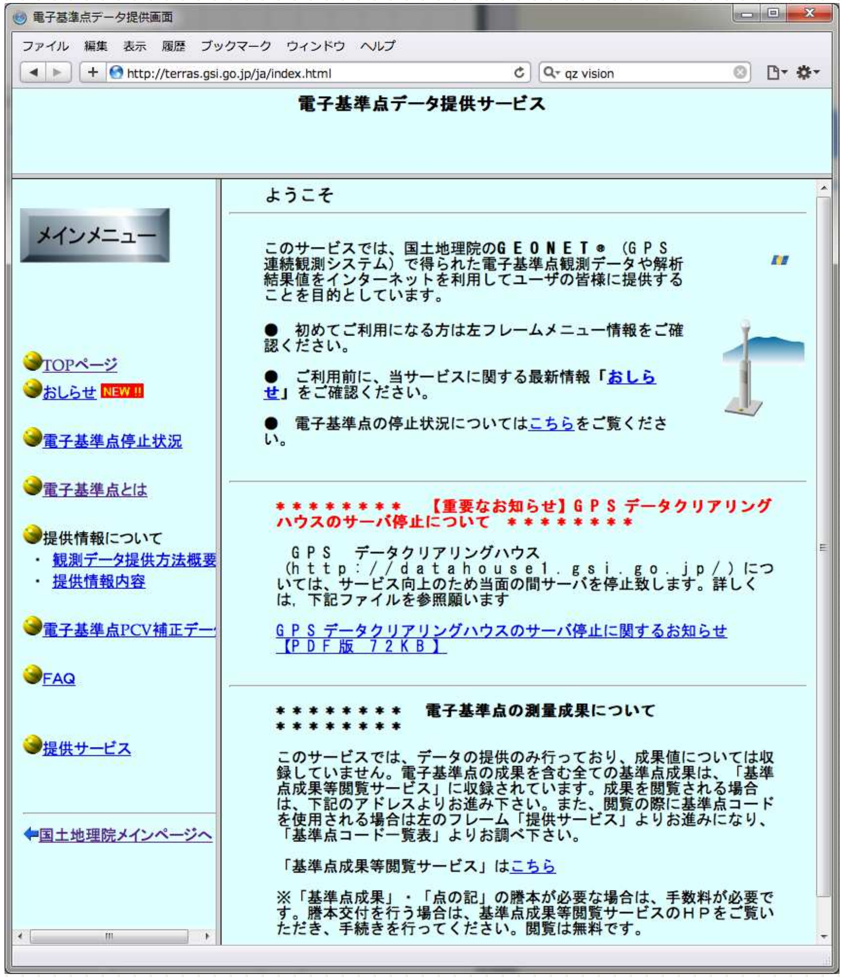
Online GNSS Data Sources

IGS Data/Products



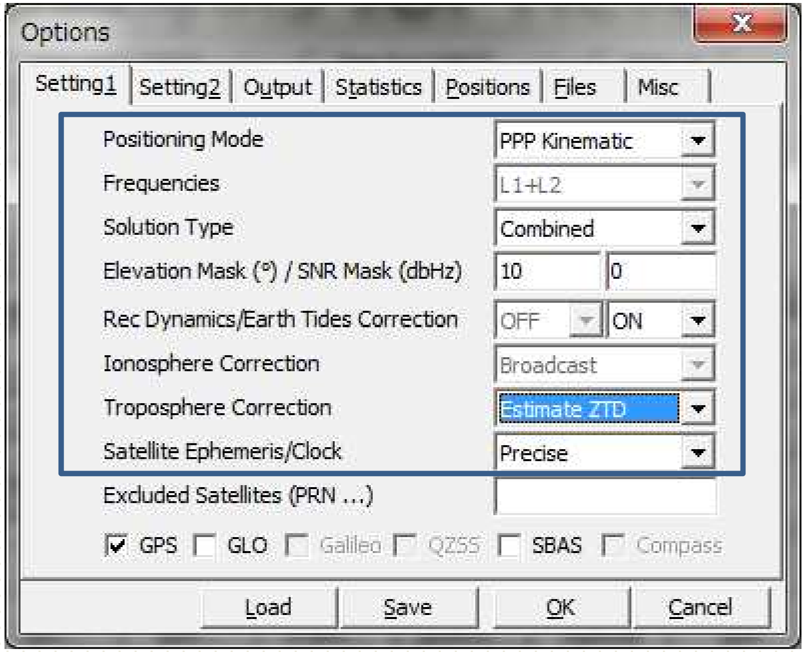
http://igscb.jpl.nasa.gov

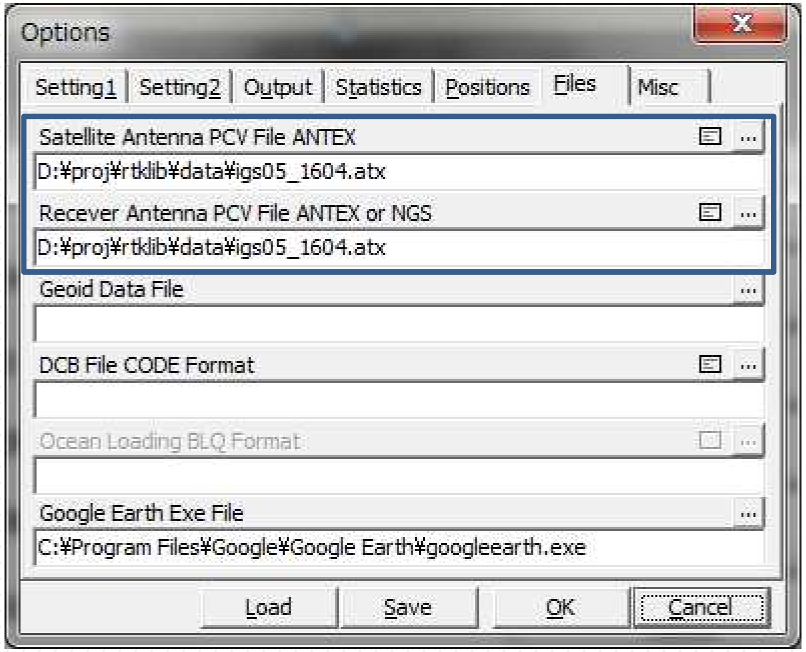
GEONET Data



http://terras.gsi.go.jp/ja/index.html

RTKPOST - Options





B-8
Advanced Topics

Advanced Topics

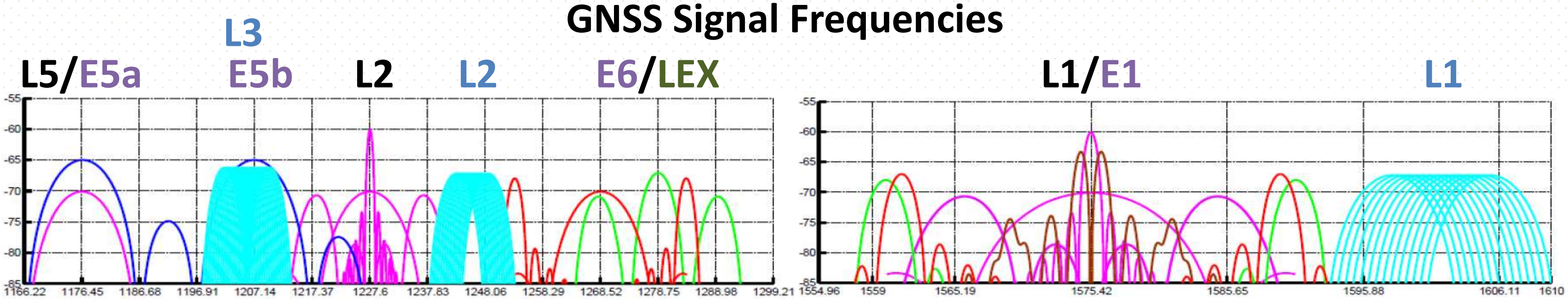
- Multi-GNSS RTK
- Long-Baseline RTK
- INS-Aided RTK
- Ambiguity Resolution for PPP
- MADOCA

GNSS Evolution

Number of Planned GNSS Satellites

System	2010	2013	2016	2019
GPS	**31**	31	32	32
GLONASS	23 (+2)	24 (+3)	24 (+3)	24 (+3)
Galileo	0	4	18	27 (+3)
Compass	6	16	30	32 (+3)
QZSS	1	1	4	7
IRNSS	0	7	7	7
SBAS	7	8	11	11
Total	**68**	**91**	**126**	**140**

GNSS Signal Frequencies



(Y.Yang, COMPASS: View on Compatibility and Interoperability, 2009)

Multi-GNSS RTK Performance

RTK Performance: Baseline 13.3 km, Instantaneous AR

		El Mask=15°				El Mask=30°			
GPS	Galileo	Fixing Ratio	RMS Error (cm)			Fixing Ratio	RMS Error (cm)		
			E-W	N-S	U-D		E-W	N-S	U-D
L1	-	49.7%	4.6	8.1	19.0	23.3%	71.4	115.0	289
L1,L2	-	99.0%	1.4	1.3	1.9	87.6%	3.4	10.5	15.5
L1,L2,L5	-	99.0%	1.4	1.3	1.9	87.3%	3.4	10.5	15.6
L1	E1	98.8%	1.3	1.2	1.9	90.1%	1.2	2.1	2.7
L1,L2	E1	98.9%	1.4	1.2	1.7	98.7%	1.2	1.0	1.6
L1,L2,L5	E1,E5a, E5b	98.9%	1.5	1.3	2.0	98.9%	1.3	1.1	1.8

GPS L1+L2

GPS+GAL L1

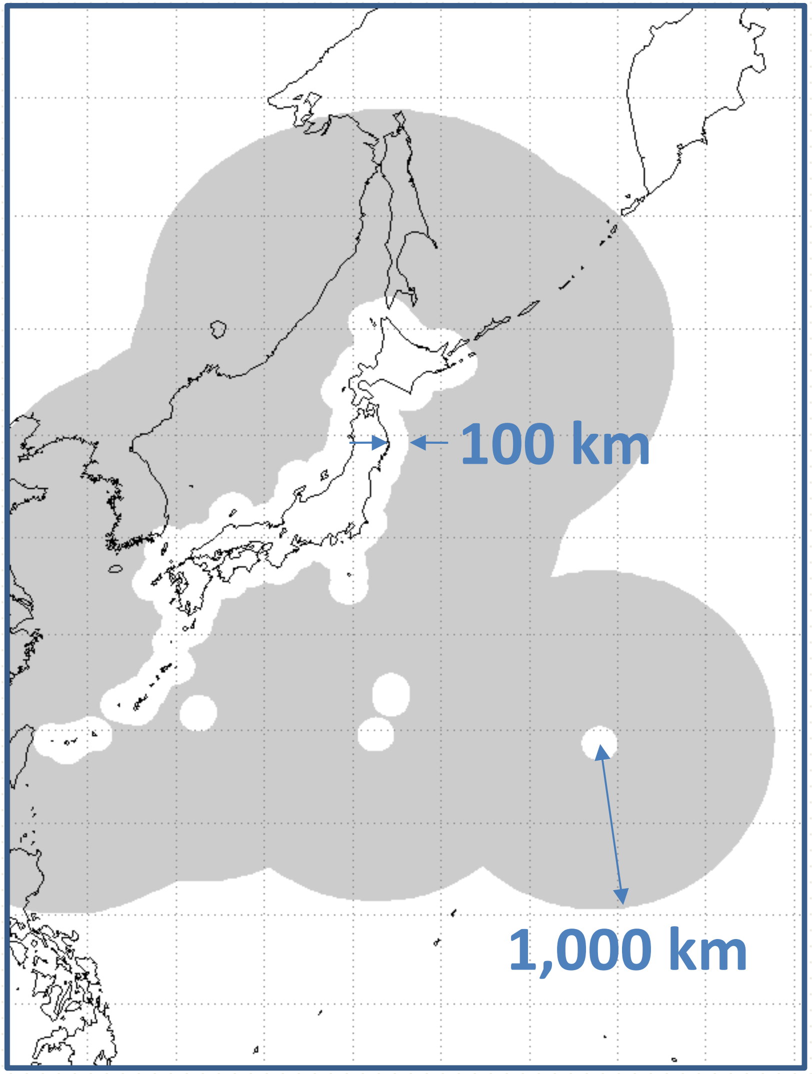
Multi-GNSS Receiver

- **Moore's Law**
  - More correlators
  - More tracking channels
  - More powerful embedded CPU
- **Consumer-grade Multi-GNSS Receiver**
  - SkyTraq: GPS + GLONASS
  - STMicro: GPS + GLONASS
  - Broadcom: GPS + GLONASS + QZSS
  - u-blox: GPS + Galileo

Issues for Multi-GNSS RTK

- **Multi-GNSS Integration Issue**
  - Time-system, Coordinate-system
  - Receiver H/W Biases
- **Multi-code System Issue**
  - L1C/A-L1P(Y)-L1Cd-L1Cp, L2P(Y)-L2C, L5I-L5Q
  - Quarter cycle phase-shift problem
- **GLONASS FDMA Issue**
  - Receiver Inter-channel biases (Receiver Interoperability)
  - Calibration Message Standard
  - Antenna Calibration

Long-Baseline RTK



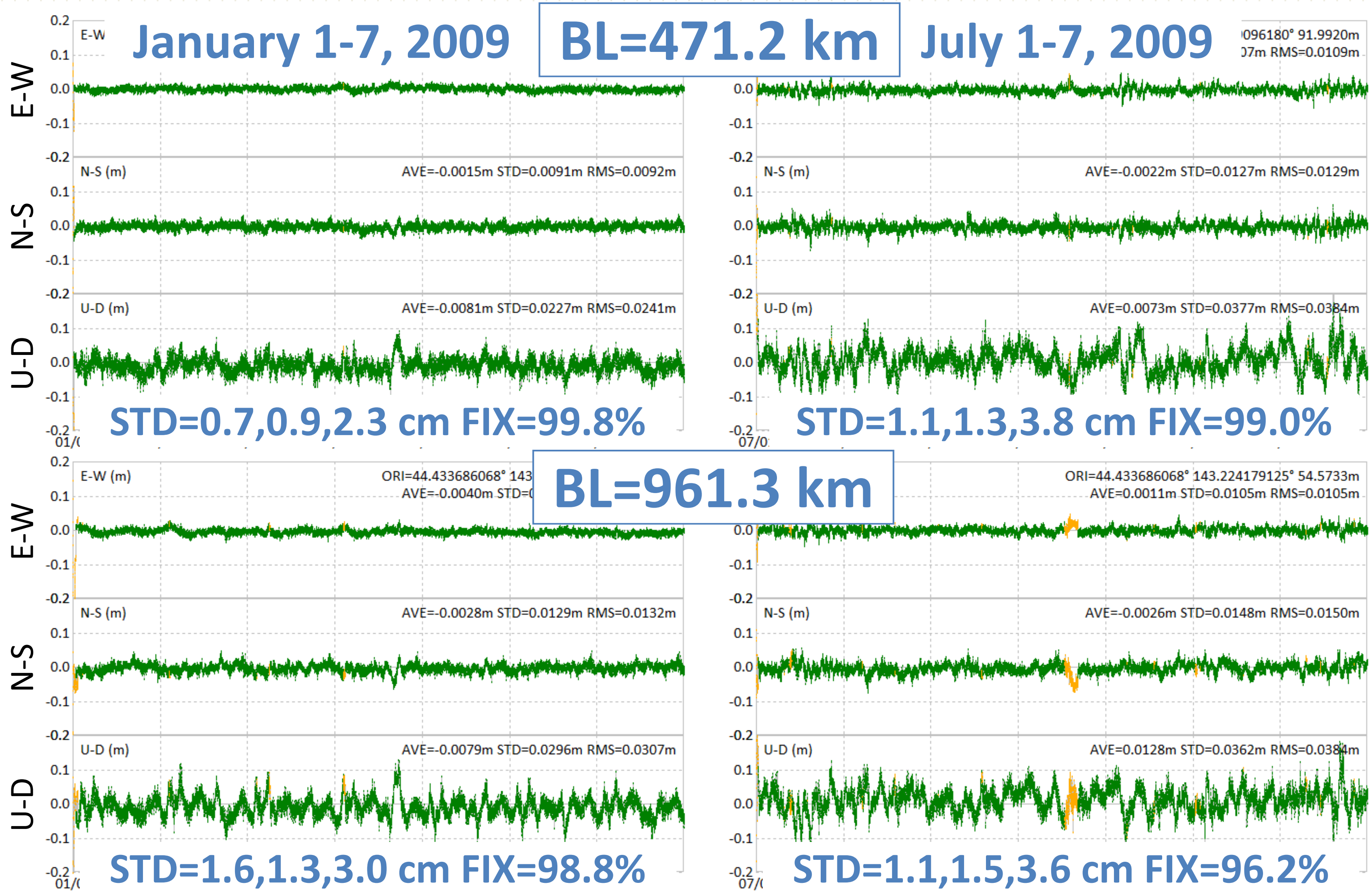
GPS Tsunami Monitoring System (Currently ~15 km off-shore)

http://www.tsunamigps.com

Long-Baseline RTK Strategy

<table>
<tr><th rowspan="2"></th><th rowspan="2">BL (km)</th><th colspan="4">Error Elimination</th><th rowspan="2">Strategy</th></tr>
<tr><th>Ephem</th><th>Ionos</th><th>Tropos</th><th>Others</th></tr>
<tr><td>S</td><td>0 – 10</td><td>Broadcast</td><td>-</td><td>-</td><td>-</td><td rowspan="2">Conventional RTK</td></tr>
<tr><td rowspan="2">M</td><td rowspan="2">10 – 100</td><td rowspan="2">Broadcast</td><td>Dual-Freq</td><td>-</td><td>-</td></tr>
<tr><td colspan="2">Interpolation</td><td>-</td><td>Network RTK</td></tr>
<tr><td>L</td><td>100 – 1,000</td><td>Real-time Precise (IGU)</td><td>Dual-Freq</td><td>Estimate ZTD + MF</td><td>Earth Tides</td><td>Long-Baseline RTK</td></tr>
<tr><td>VL</td><td>>1,000</td><td>Non-RT Precise (IGR, IGS)</td><td>Dual-Freq</td><td>Estimate ZTD + MF</td><td>Earth Tides, Ph-WU</td><td>Post-Processing or PPP</td></tr>
</table>

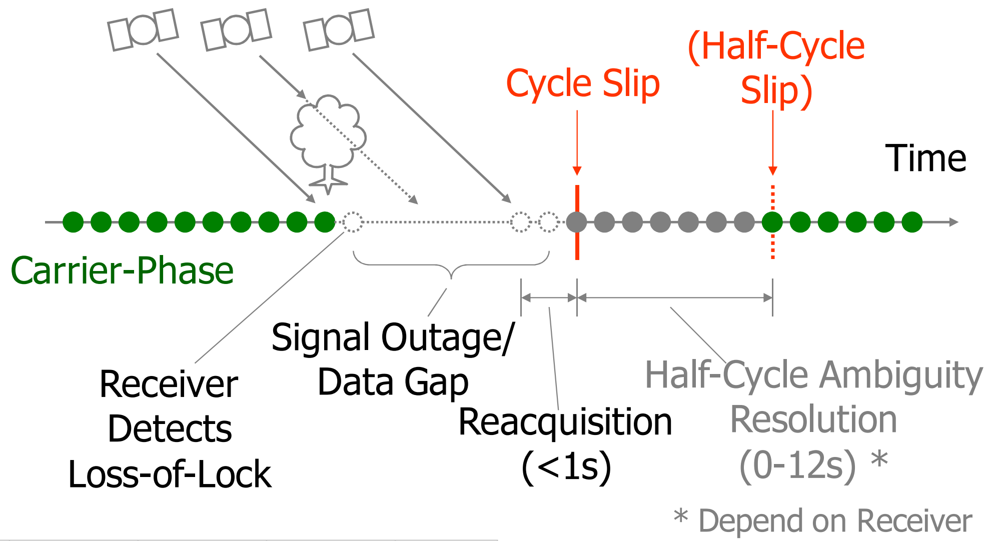
Long-Baseline RTK with RTKLIB

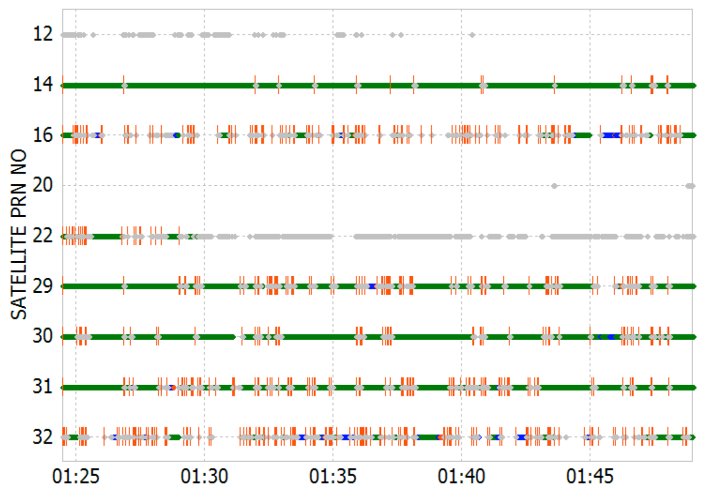


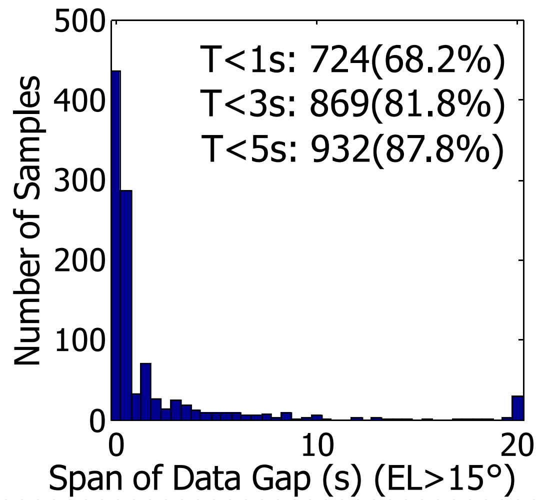
Mobile AP issues for RTK

- **Cycle-Slips**
  - Frequent cycle-slip with around obstacles
  - Miss-detection of cycle-slip
- **Low Solution Availability**
  - Long acquisition time by weak signal (Low C/N0)
  - Half-cycle ambiguity resolution with Costas-PLL
  - Low fixing ratio
- **High Noise Level**
  - High multipath level even in carrier-phase
  - Jamming by RFI

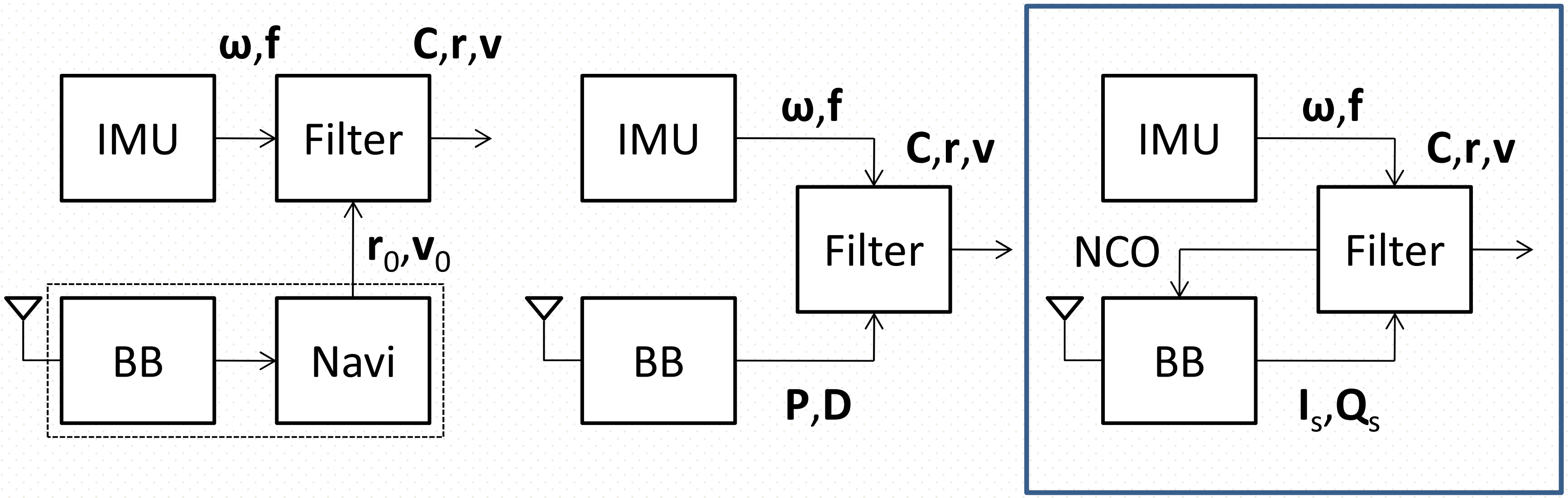
Cycle-Slips


Cycle Slip
(Half-Cycle Slip)
Time
Carrier-Phase
Signal Outage/ Data Gap
Receiver Detects Loss-of-Lock
Reacquisition (<1s)
Half-Cycle Ambiguity Resolution (0-12s) *
* Depend on Receiver


SATELLITE PRN NO
12
14
16
20
22
29
30
31
32
01:25
01:30
01:35
01:40
01:45


T<1s: 724(68.2%)
T<3s: 869(81.8%)
T<5s: 932(87.8%)
Number of Samples
Span of Data Gap (s) (EL>15°)
0
100
200
300
400
500
10
20

INS-Aided RTK



Loosely-Coupled Integration

Tightly-Coupled Integration

Deep Integration (Ultra-Tightly)

High sensitivity (DLL, PLL)

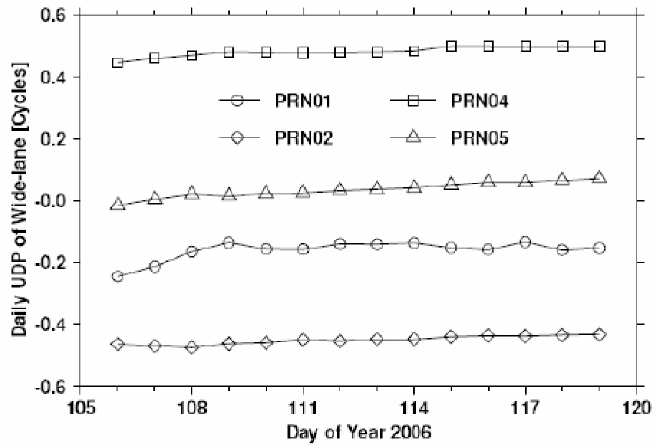
Slip resistance

Ambiguity Resolution for PPP

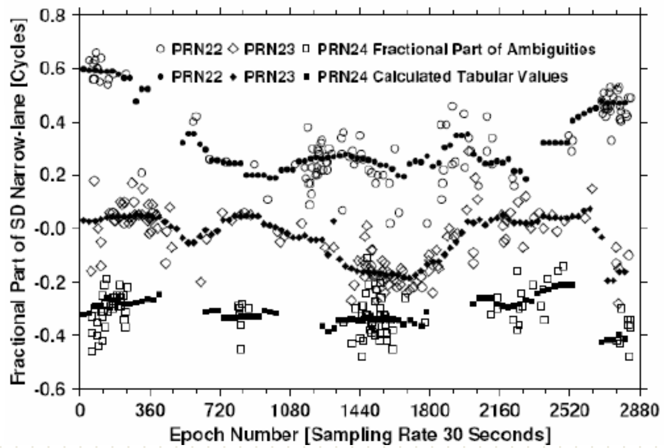
- **with AR for PPP**
  - Improve Convergence Time
  - Improve Accuracy of Static Solution (EW, UD)
  - Improve Stability of Kinematic Solution
- **Difficulties of AR for PPP**
  - Unknown Satellite Initial Phase Biases
  - Effect of Precise Orbit/Clock Error
  - Effect of Ionospheric Delay
  - Code/Phase Bias Instability
  - Multipath Effect at Reference Station Network

M.Ge et al., EGU 2007

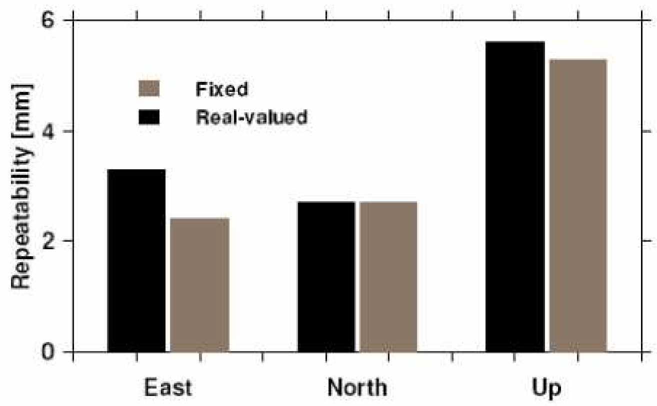
WL Phase Bias Stability



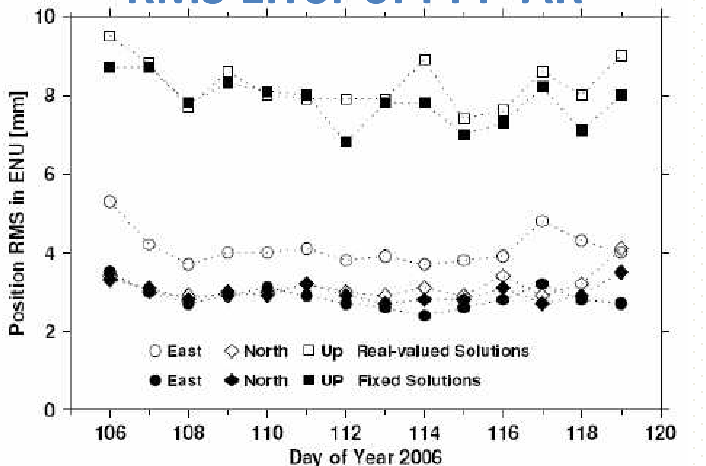
NL Phase Bias Stability



Repeatability of PPP-AR



RMS Error of PPP-AR



M.Ge et al., Resolution of GPS carrier-phase ambiguity in precise point positioning, EGU Assembly 2007

D.Laurichesse, ION 2010

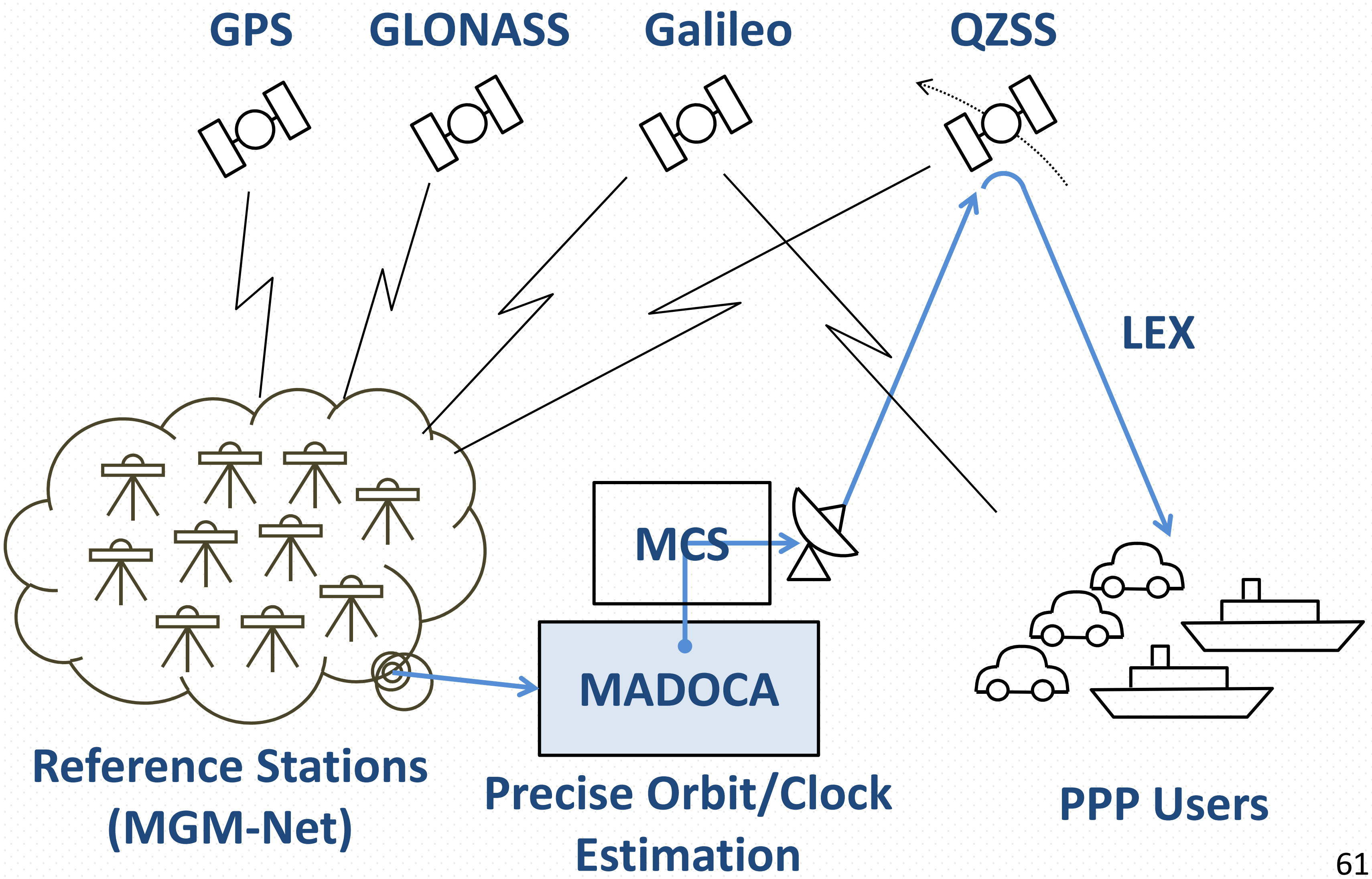
- **Real-Time Implementation of PPP-AR**
  - Network WL ambiguity fixing
  - Parameter estimation by EKF with iono-free code/phase: phase-clock, code-phase-bias, ZTD, station position, orbit correction to IGU, phase ambiguity
  - Orbit construction + high-rate clock generation
- **Evaluation of Accuracy**
  - Orbit: 4cm, code-clock: 5 cm, phase-clock: 1cm
- **RT-PPP with AR ("CNES Integer PPP")**
  - 1 cm HRMS

MADOCA

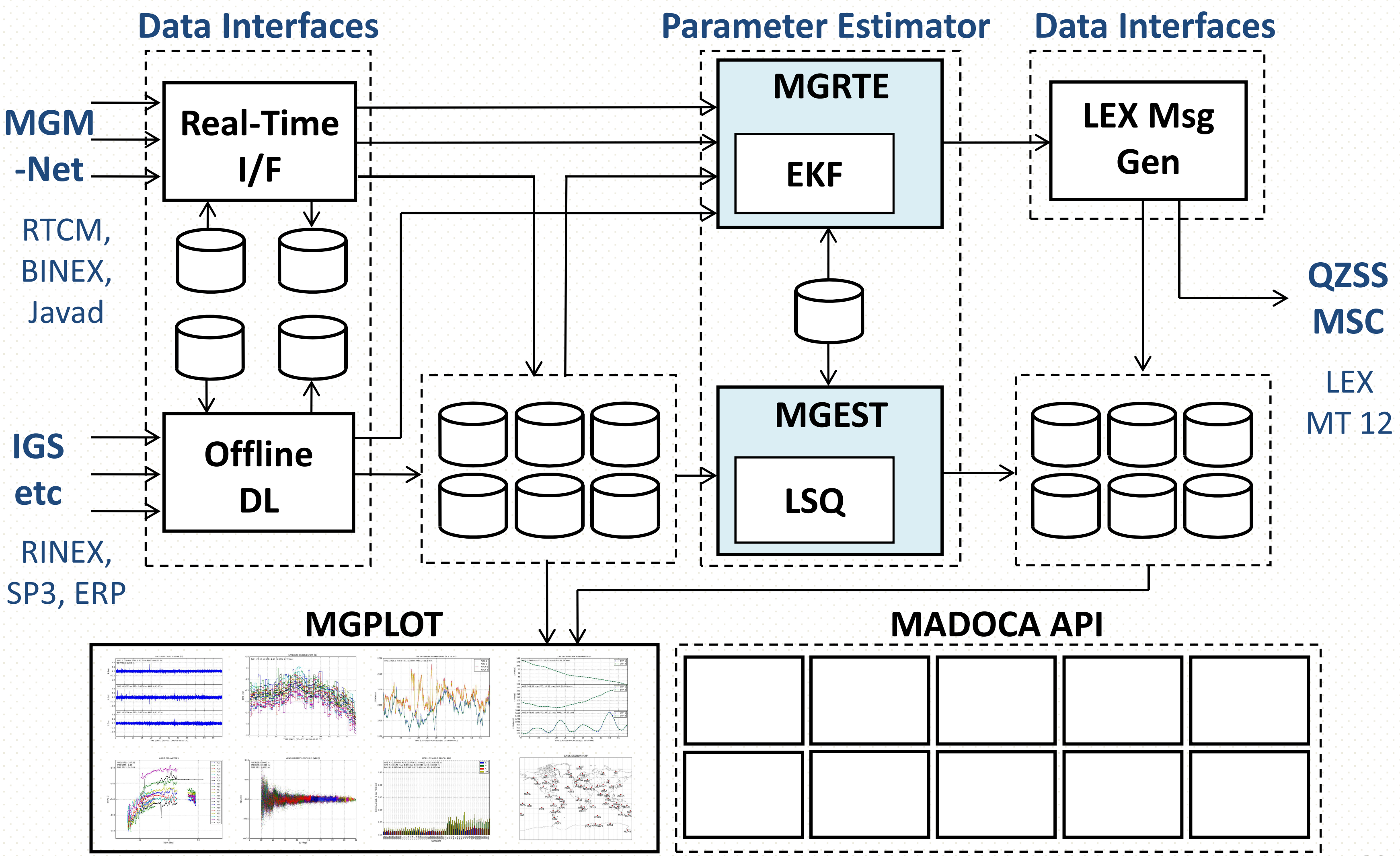
Multi-GNSS Advanced Demonstration tool for Orbit and Clock Analysis

- **For real-time PPP service via QZSS LEX**
  - Many (potential) applications over global area
- **Precise orbit/clock for multi-GNSS constellation**
  - Key-technology for future cm-class positioning
- **Brand-new codes developed from scratch**
  - Optimized multi-threading design for recent CPU
  - As basis of future model improvements

Real-Time PPP via QZSS



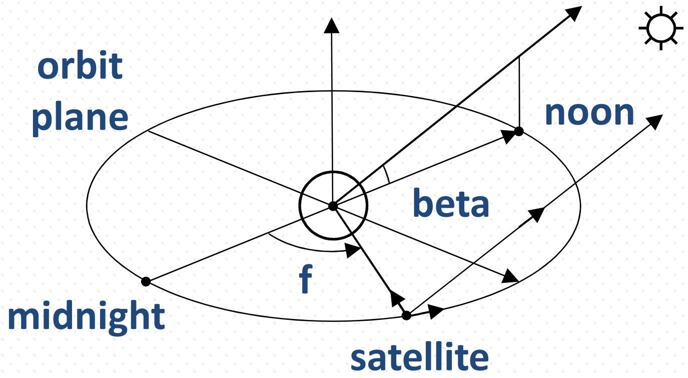
MADOCA Architecture

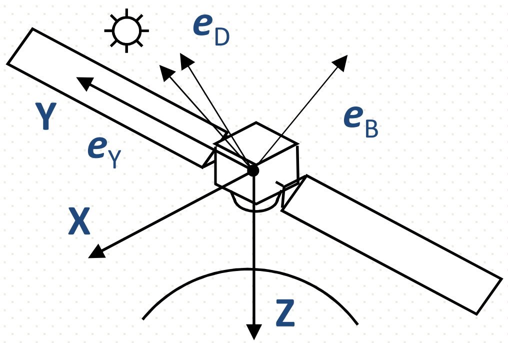


MADOCA: Models

- **Satellite Orbit Models**
  - EGM 2008+solid earth tide+FES2004
  - Sun, Moon, Venus and Jupiter with JPL DE421
  - Empirical SRP model, ...
- **Measurement Models**
  - ZD Iono-free phase+ pseudorange, 2nd-order-iono
  - ZTD+gradient estimation with GPT+GMF/VMF1
  - IERS DEHANTIDEINEL+FES2004+pole tide+CMC
- **ECI-ECEF Coordinates Transformation**
  - IAU 2000A/2006 by IAU SOFA

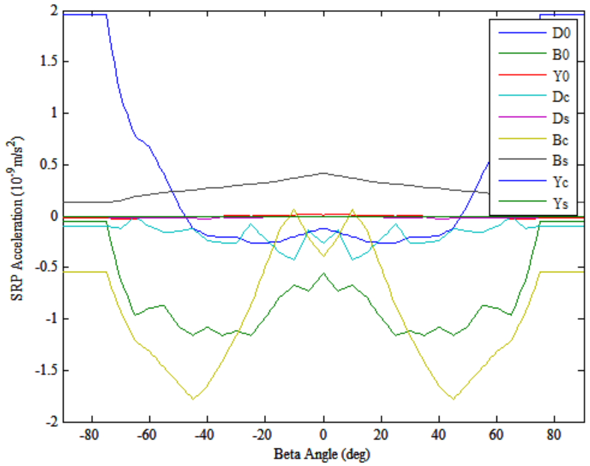
MADOCA: SRP Model



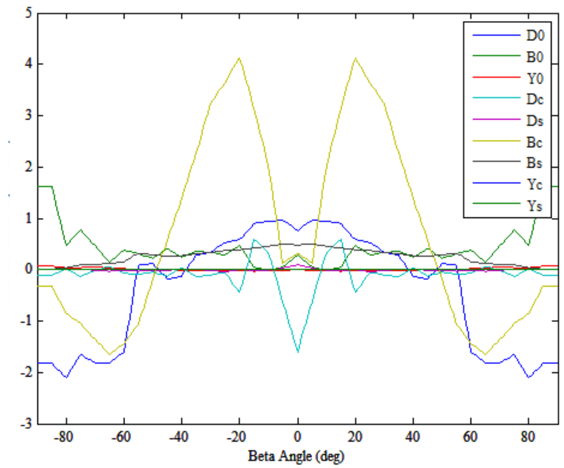


$$\boldsymbol{a}_{srp} = S\,((D_0 + D_C \cos f + D_S \sin f)\,\boldsymbol{e}_D + (B_0 + B_C \cos f + B_S \sin f)\,\boldsymbol{e}_B + (Y_0 + Y_C \cos f + Y_S \sin f)\,\boldsymbol{e}_Y) \times 10^{-9}\ (m/s^2)$$

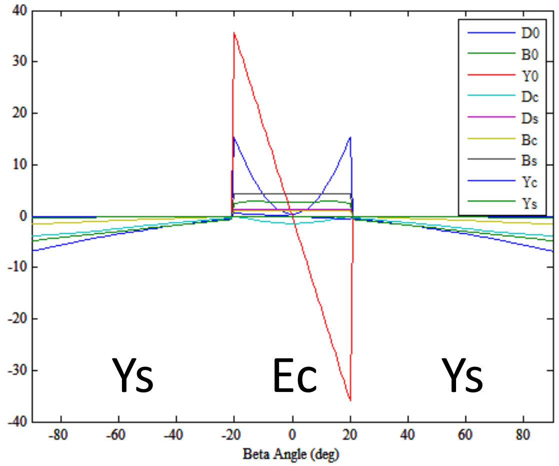
GPS Block IIR



GLONASS



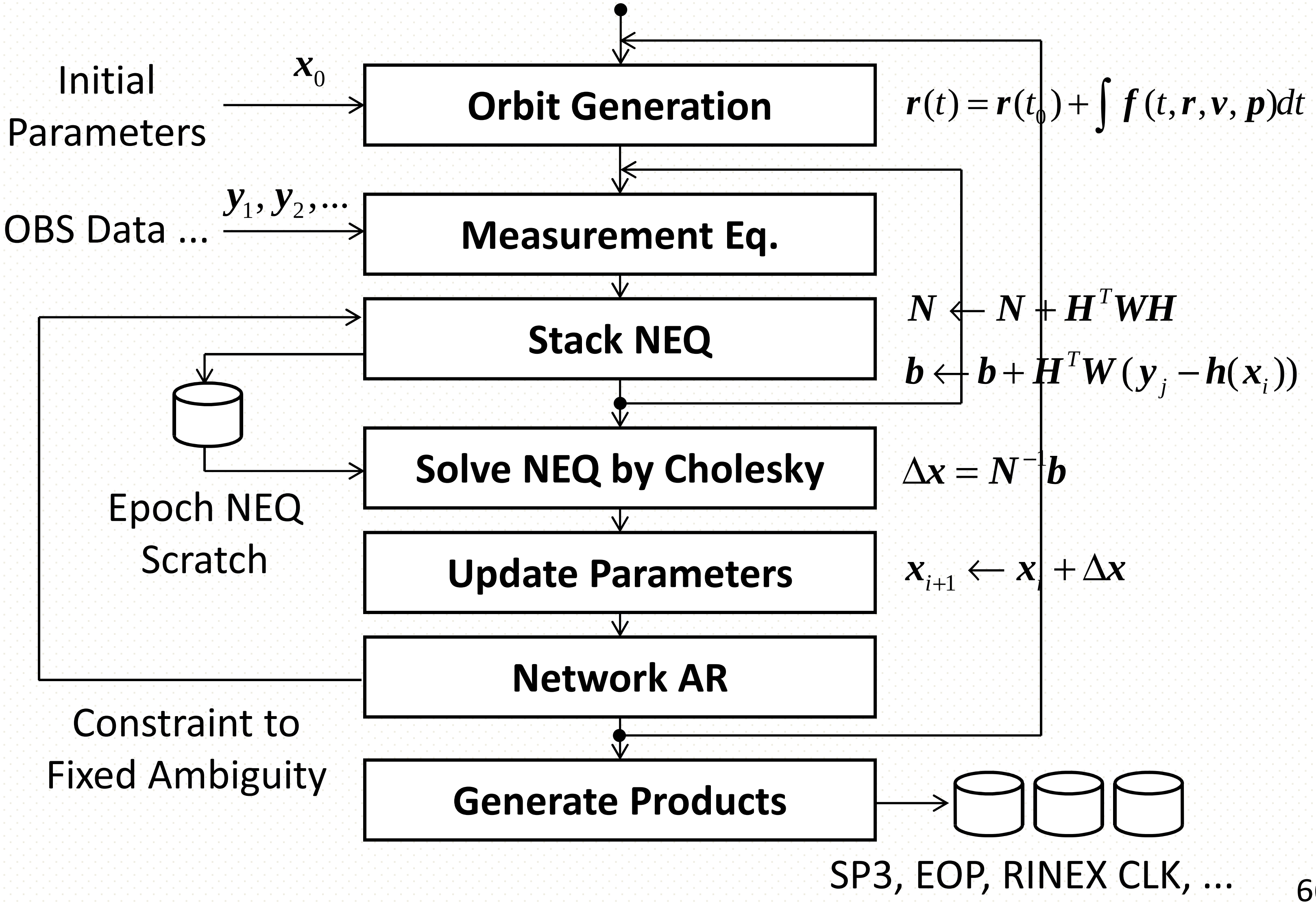
QZSS



MADOCA: Parameter Adjustment

	Offline	Real-Time
Algorithm	**Iterated Weighted LSQ**	**Dual-Cycle-EKF**
Estimated Parameters	Orbit, SRP/Emp-Acc, Clock, Position, ZTD/Grad, Ambiguity, Bias, EOP	
Measurements	ZD Carrier-Phase and Peudo-range	
Numerical Solver	NEQ by Cholesky Factorization	**Numerical Stable EKF**
Clock Estimation	**Parameter Elimination in NEQ**	State as White-Noise or Random-Walk
Integer Ambiguity Resolution	**Network AR** (Ge., 2005)	Real-Time Network AR

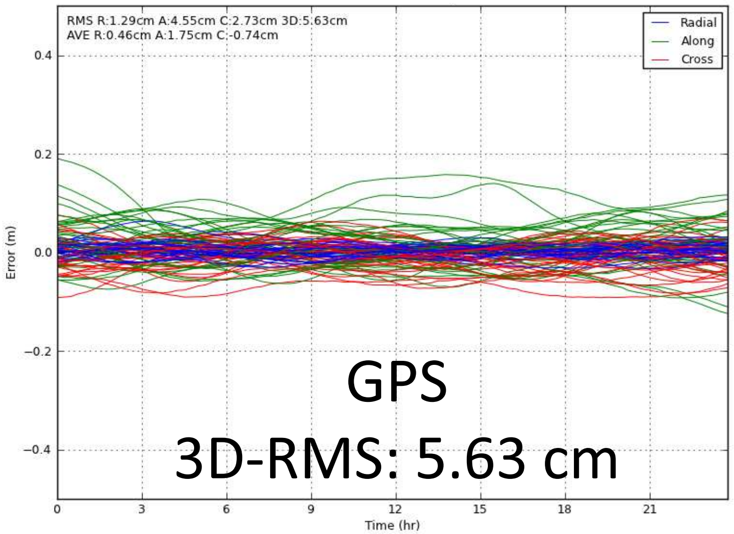
MADOCA: Iterated LSQ



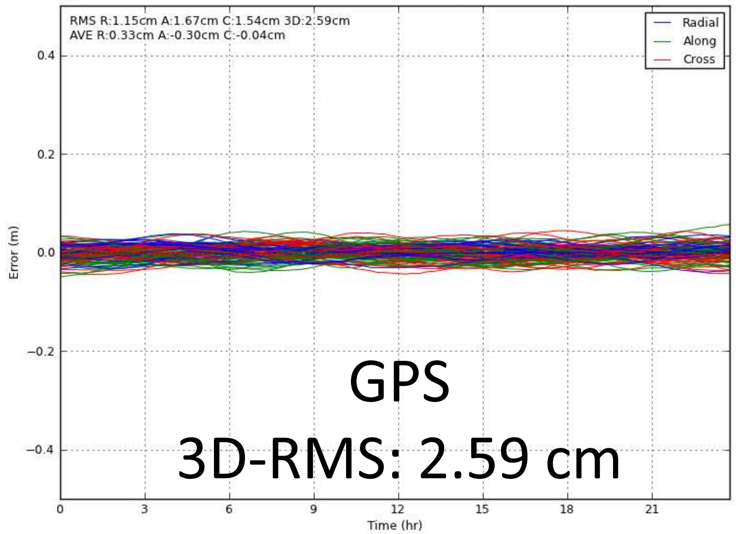
Network AR

- Dynamic baseline selection to convert ZD to DD
- WL and NL DD ambiguities by rounding
- Validation by confidence function and FCB
- For GPS, QZSS and Galileo (no GLONASS)

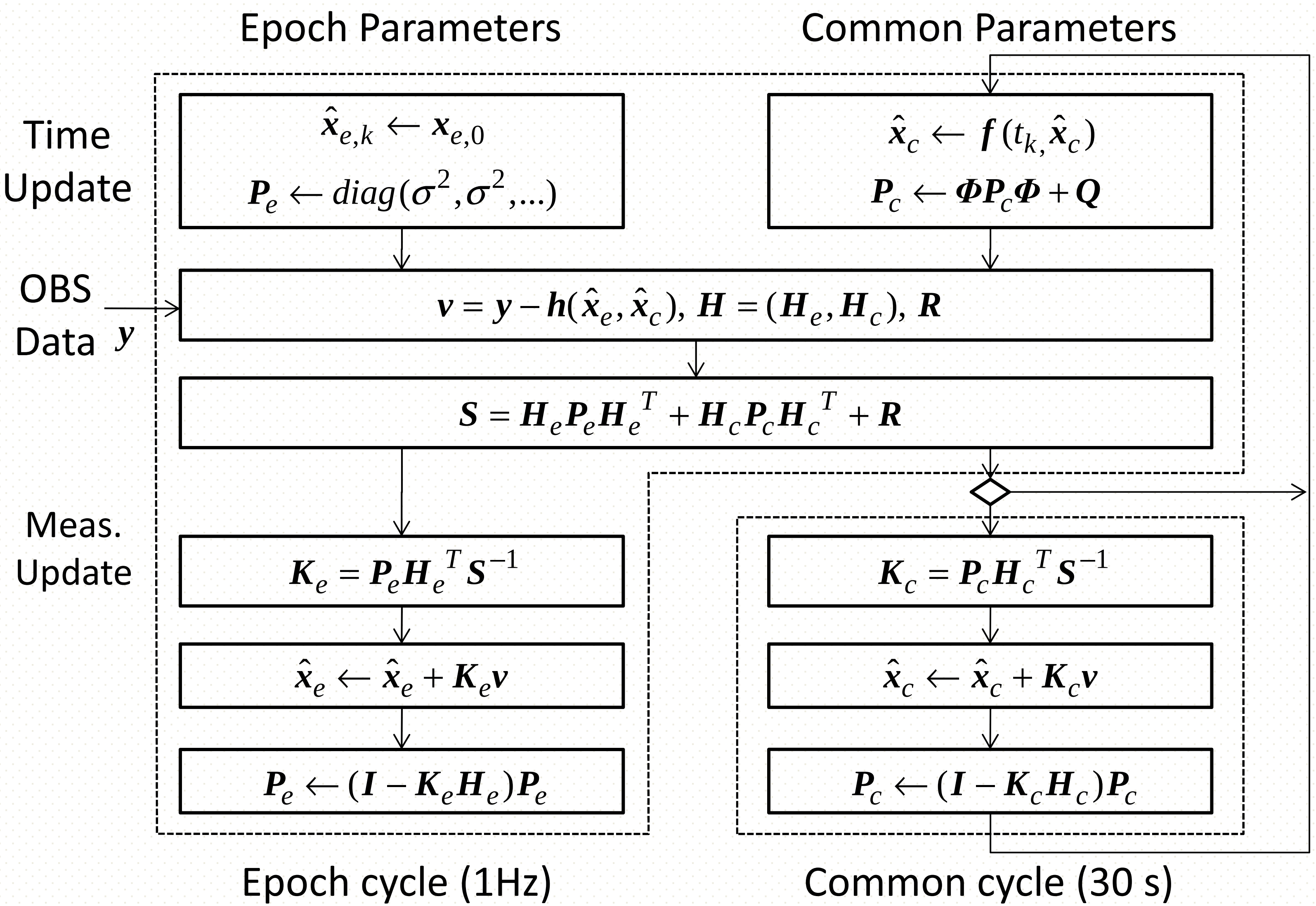
AR-OFF



AR-ON



Dual-Cycle-EKF



Numerically Stable EKF

Measurement Update of EKF

$$K = P^- H^T (HP^- H^T + R)^{-1}$$

$$x^+ = x^- + K(y - h(x^-))$$

$$P^+ = (I - KH)P^-$$

Standard EKF

(1) **v** = **y** - **h**(**x**), **H**, **R**	
(2) **D** = **P H**^{T}	(sparse)
(3) **S** = **H D** + **R**	(sparse)
(4) **U** = chol(**S**)	DPOTRF
(5) **K** = (**D U**$^{-1}$) **U**$^{-T}$	DTRSM
(6) **x** = **x** + **K v**	DGEMV
(7) **P** = **P** - **K D**^{T}	DGEMM

Numerically Stable EKF

(1) **v** = **y** - **h**(**x**), **H**, **R**	
(2) **D** = **P H**^{T}	(sparse)
(3) **S** = **H D** + **R**	(sparse)
(4) **U** = chol(**S**)	DPOTRF
(5) **E** = **D U**$^{-1}$	DTRSM
(6) **K** = **E U**$^{-T}$	DTRSM
(7) **x** = **x** + **K v**	DGEMV
(8) **P** = **P** - **E E**^{T}	DSYRK

Offline GPS/GLO Orbit

GPS (32 sats)

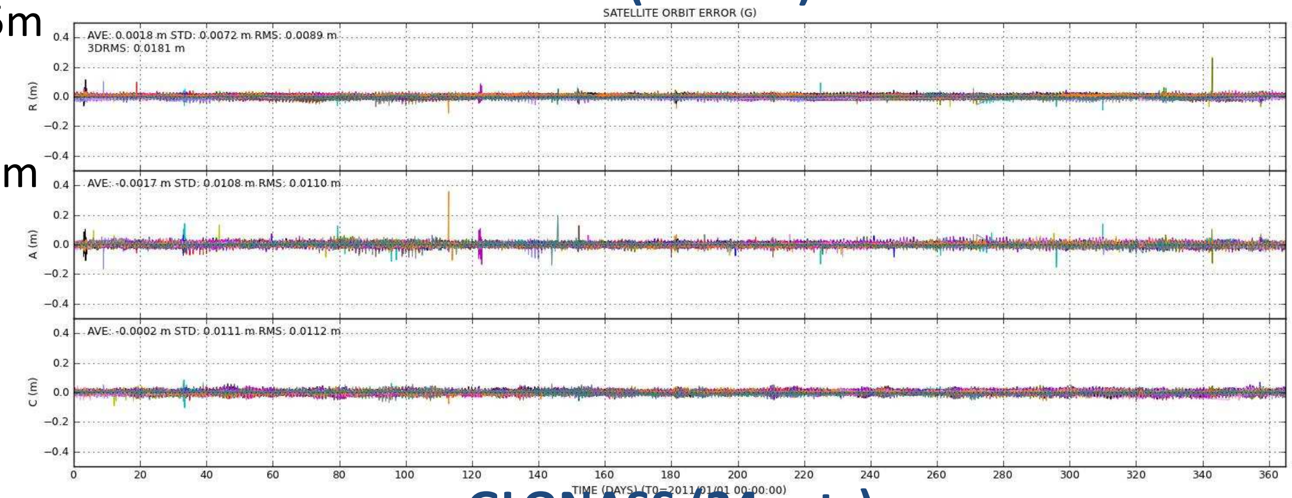
+0.5m

R

-0.5m

A

C



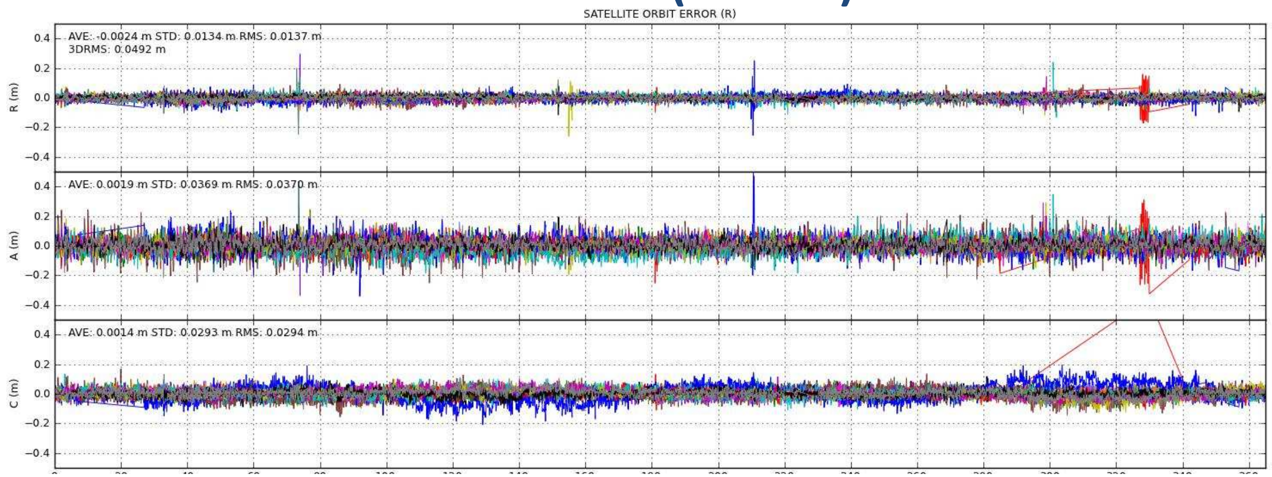
RMS
R: 0.89 cm
A: 1.10 cm
C: 1.12 cm
3D: 1.81 cm

GLONASS (24 sats)

R

A

C



RMS
R: 1.37 cm
A: 3.70 cm
C: 2.94 cm
3D: 4.92 cm

2011/01/01 - 2011/12/31 (365 days), wrt IGS Final

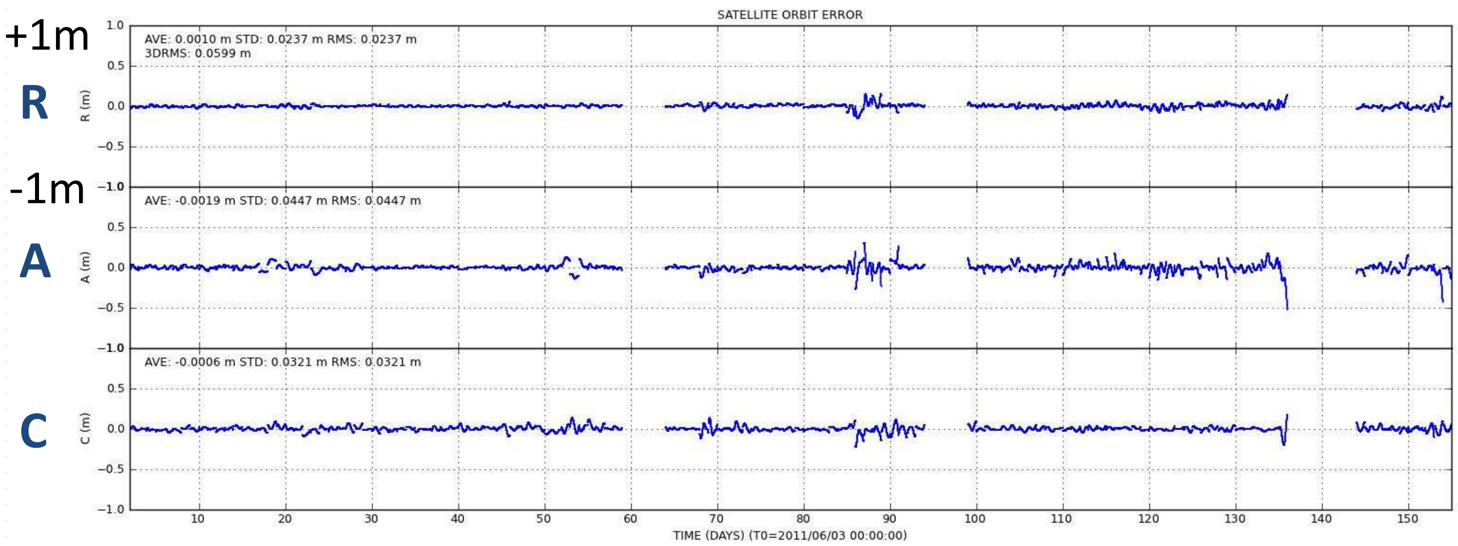
MADOCA GPS Orbit vs. IGS AC

IGS AC	Analysis Software	# of Stas	GPS Orbit RMS (cm)				Clock (ns)	
			R	A	C	3D	STD	RMS
	MADOCA 0.3.0	**77**	**0.89**	**1.10**	**1.12**	**1.81**	**0.109**	**0.131**
ESA	NAPEOS 3.5	110	0.97	1.33	1.09	1.98	0.116	0.183
CODE	Bernese 5.1	231	1.01	1.36	1.14	2.04	0.075	0.089
NGS	arc, orb, pages, gpscom	199	0.95	1.46	1.41	2.24	-	-
GFZ	EPOS.P.V2	191	1.15	1.64	1.59	2.56	0.146	0.169
MIT	GAMIT 10.33, GLOBK 5.16	263	1.37	2.12	1.39	2.88	0.277	0.316
NRCan	GIPSY/OASIS-II 5.0	91	2.58	1.72	1.77	3.57	0.128	0.148
JPL	GIPSY/OASIS-II 5.0	142	2.62	1.67	1.98	3.68	0.168	0.226
SIO	GAMIT 10.20, GLOBK 5.08	258	2.42	2.26	1.77	3.75	-	-
GRG	GINS, DYNAMO	134	2.47	2.80	1.74	4.12	0.172	0.212

2011/01/01 -2011/12/31 (365 days), wrt IGS Final

Offline QZSS Orbit

QZSS-1 Michibiki J01

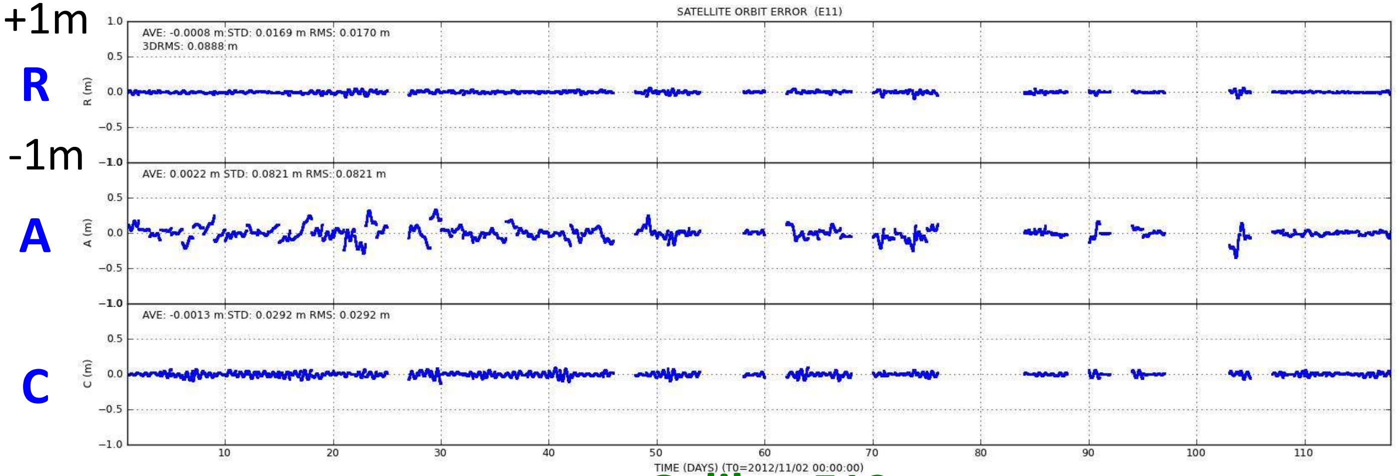


RMS
R: 2.37 cm
A: 4.47 cm
C: 3.21 cm
3D: 5.99 cm

2011/06/04 - 2011/11/03 (153 days), 24 H-overlap

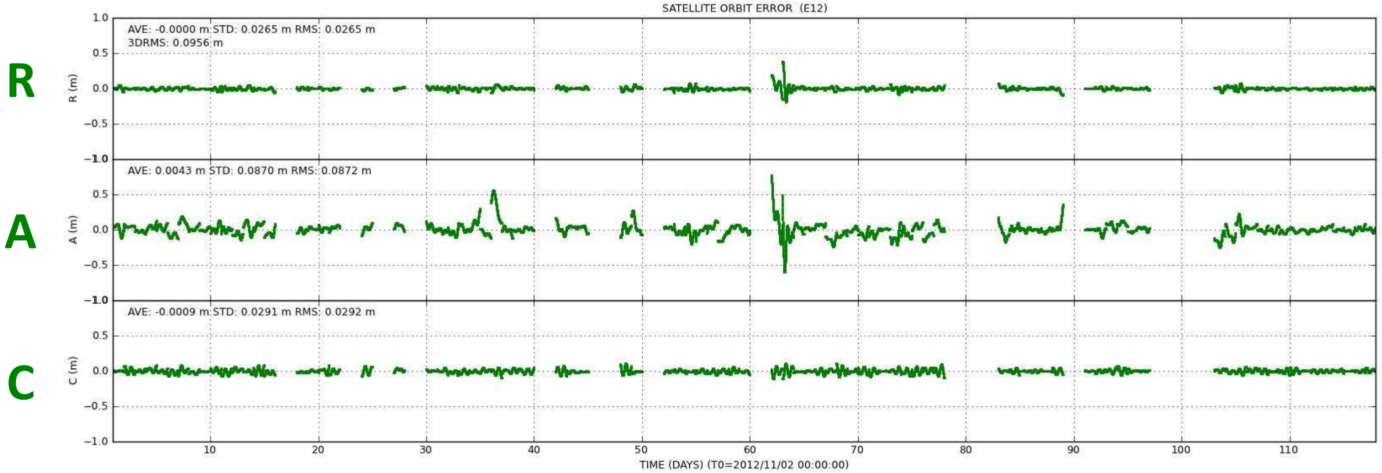
Offline Galileo Orbit

Galileo E11



RMS
R: 1.70 cm
A: 8.21 cm
C: 2.92 cm
3D: 8.88 cm

Galileo E12

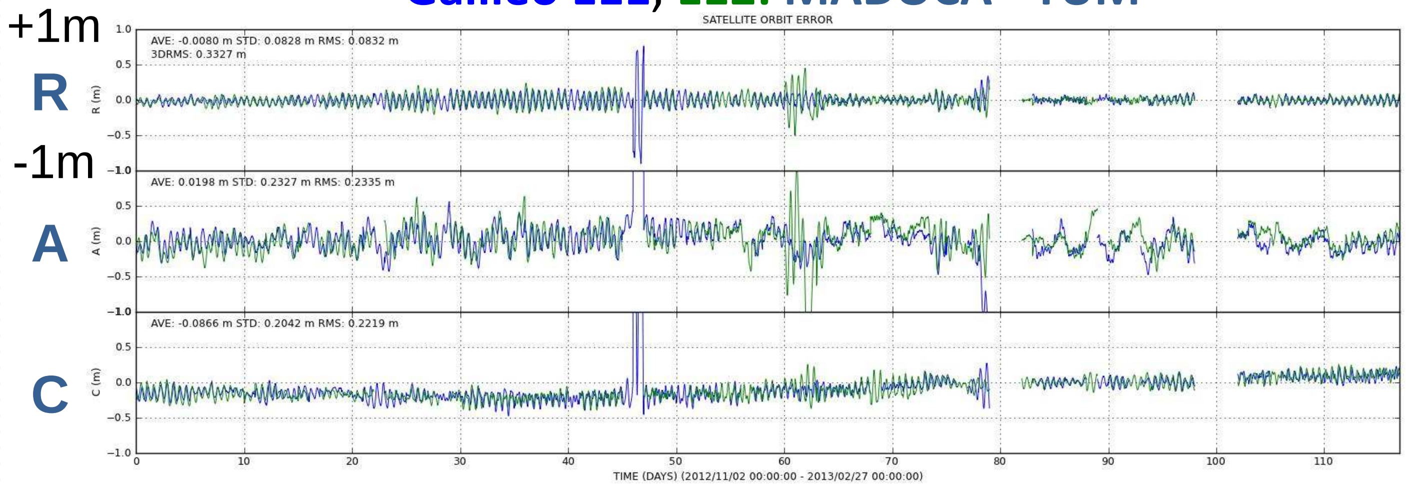


RMS
R: 2.65 cm
A: 8.72 cm
C: 2.92 cm
3D: 9.56 cm

2012/11/2 - 2013/02/27 (117 days), 24H-overlap

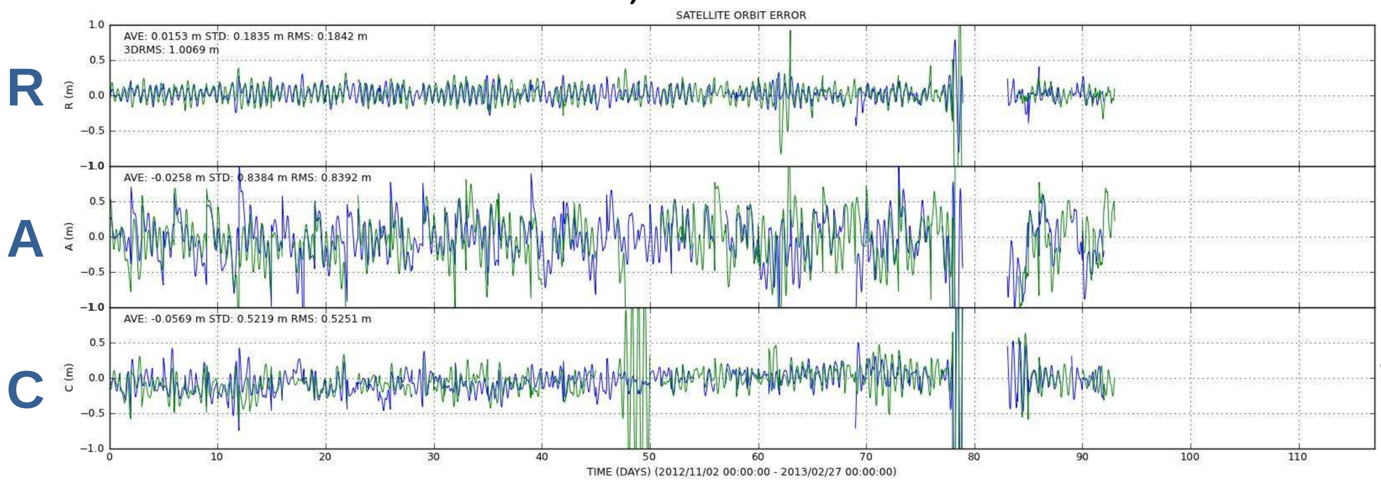
Galileo Orbit vs. TUM/GRM

Galileo E11, E12: MADOCA - TUM



RMS
R: 8.32 cm
A: 23.35 cm
C: 22.19 cm
3D: 33.27 cm

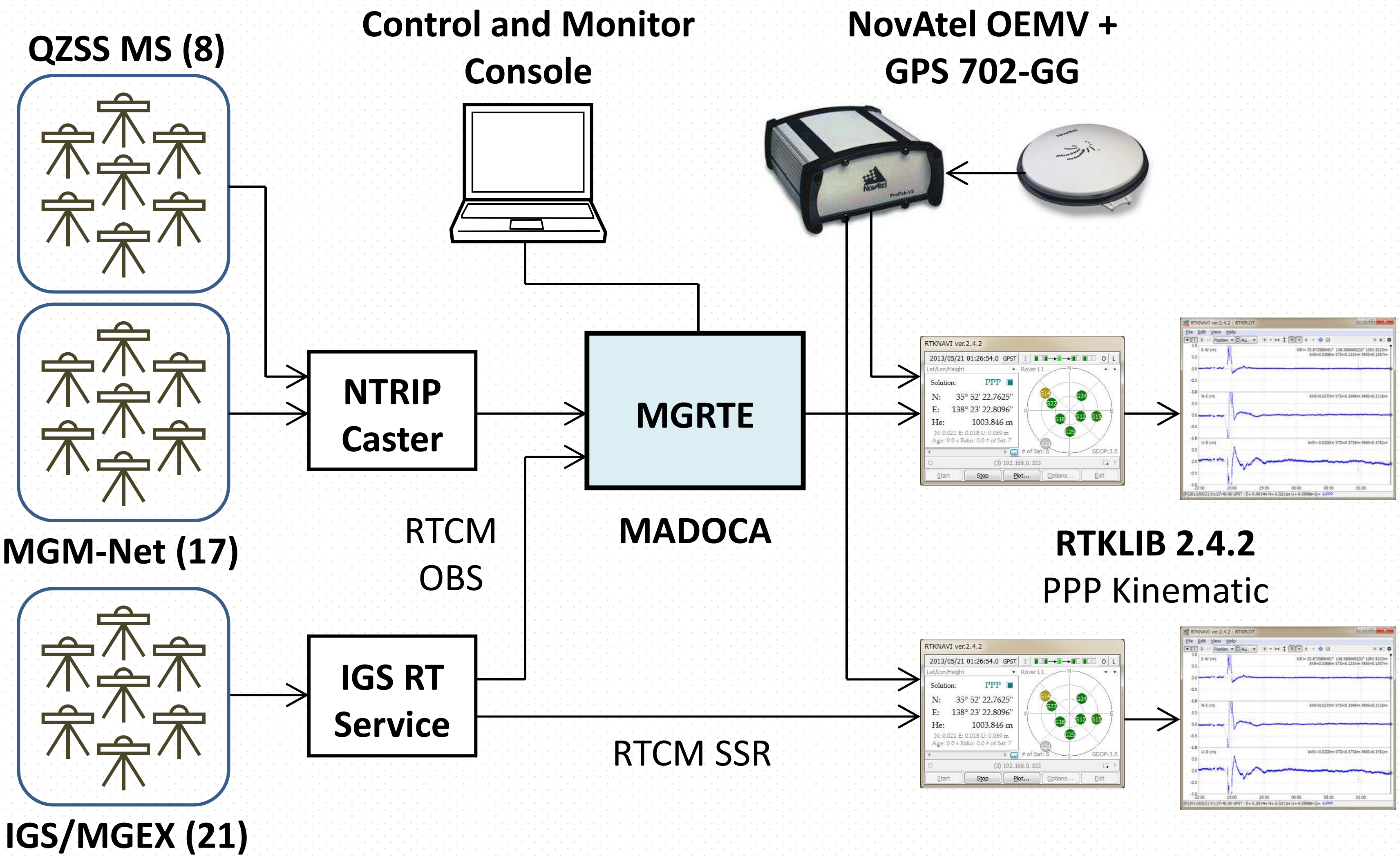
Galileo E11, E12: MADOCA - GRM



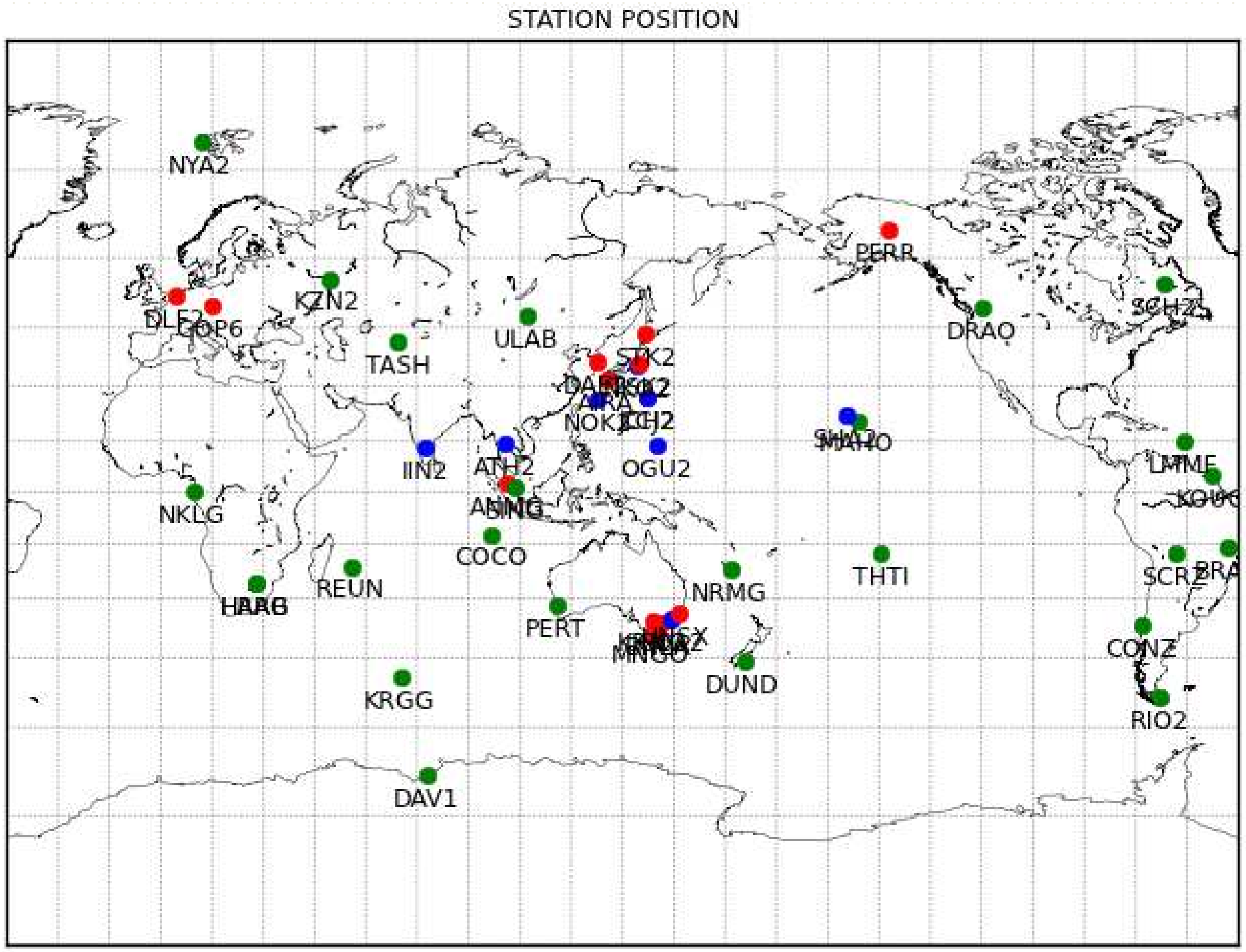
RMS
R: 18.42 cm
A: 83.92 cm
C: 52.51 cm
3D: 100.69 cm

2012/11/02 - 2013/02/27 (117 days)

Real-Time PPP Test



Reference Station Network

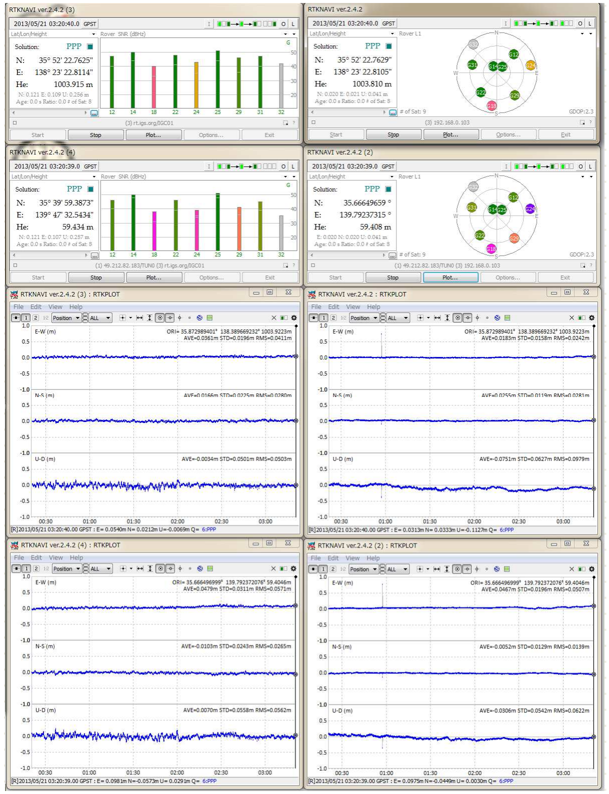


● QZSS-MS (8) ● MGM-Net (17) ● IGS/MGEX (21)

Test Snapshots

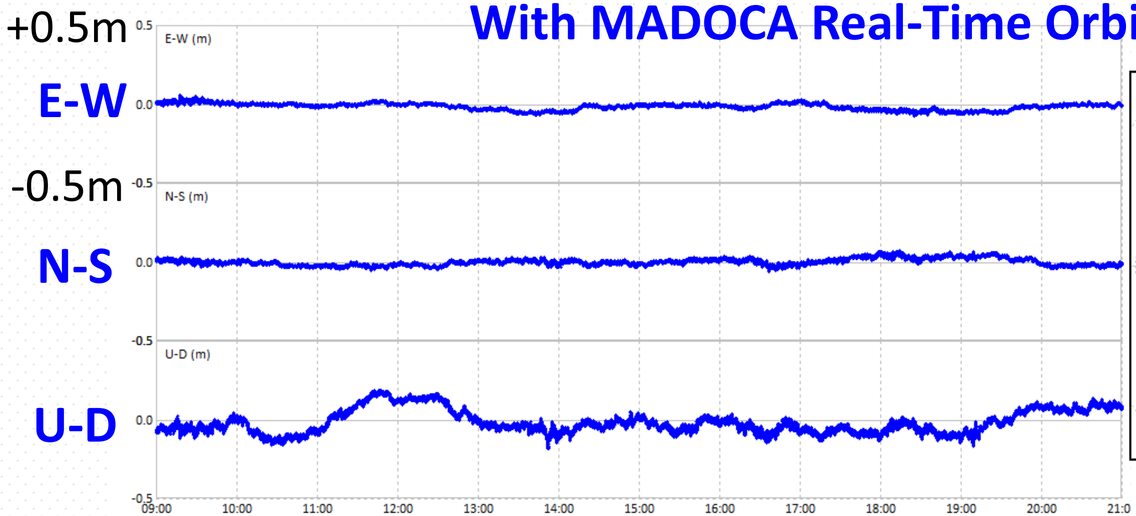
Control and Monitor Console of MADOCA MGRTE

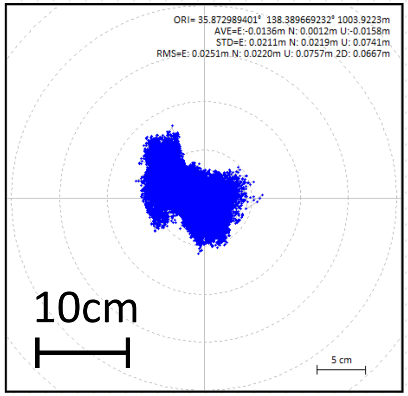
Real-time PPP by RTKLIB 2.4.2



Real-Time PPP Test Results

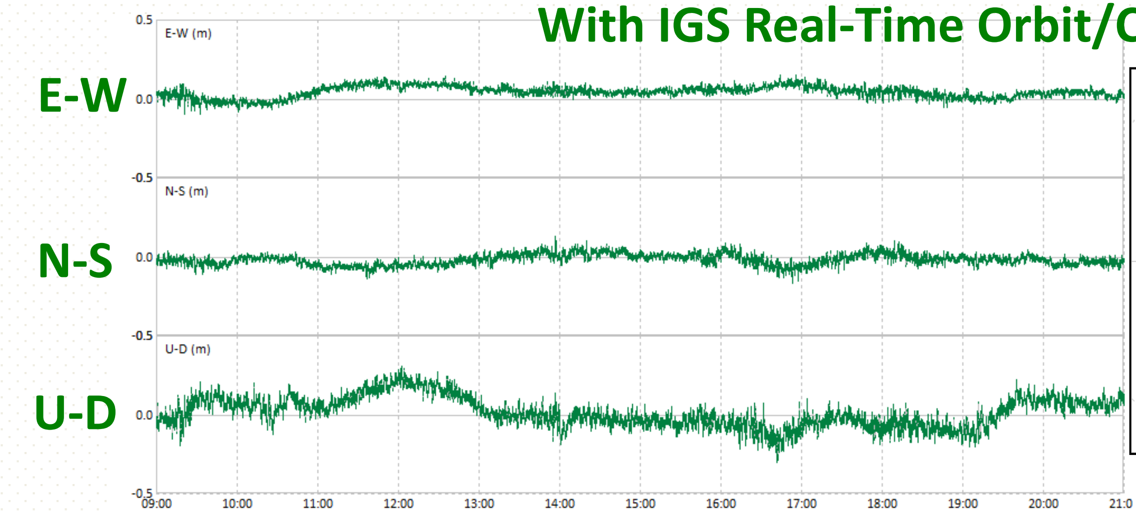
With MADOCA Real-Time Orbit/Clock

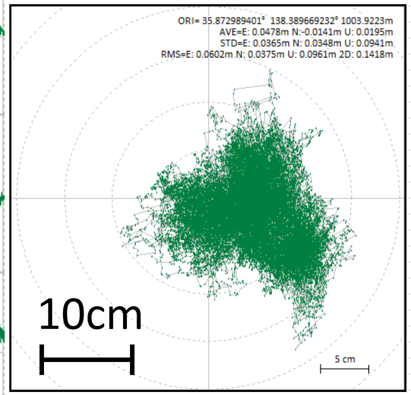




RMS
E: 2.51 cm
N: 2.20 cm
U: 7.57 cm

With IGS Real-Time Orbit/Clock





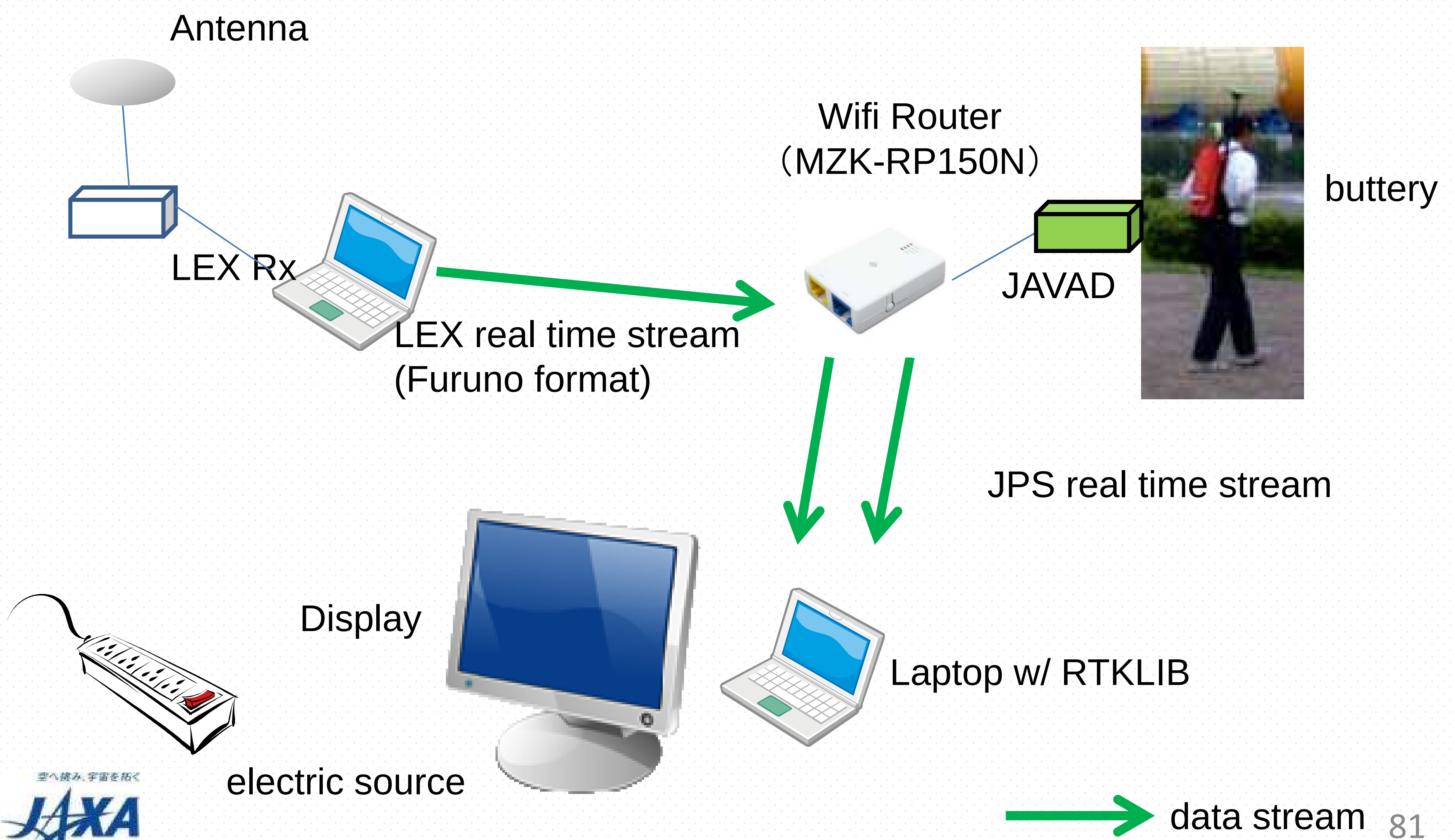
RMS
E: 6.02 cm
N: 3.75 cm
U: 9.61 cm

2013/05/20 09:00 - 21:00 (12 h), 1 Hz, only with GPS

B-9
JAXA Activities by Tateshita-san

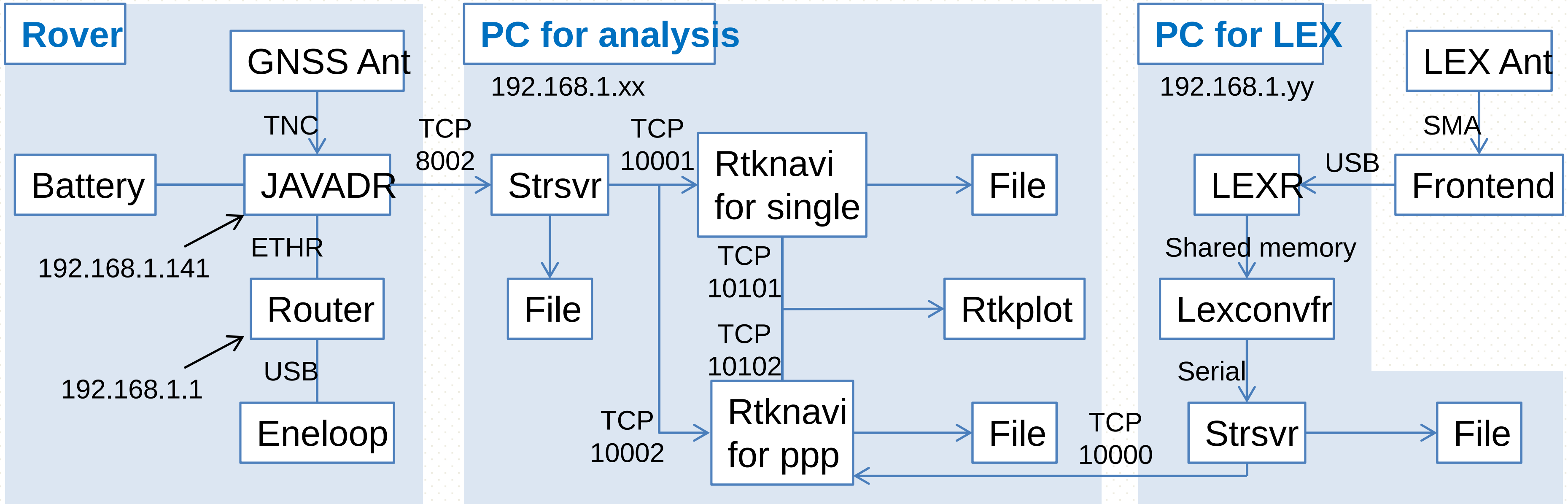
B-10
QZSS LEX-PPP Demonstration
(provided by JAXA)

fine day

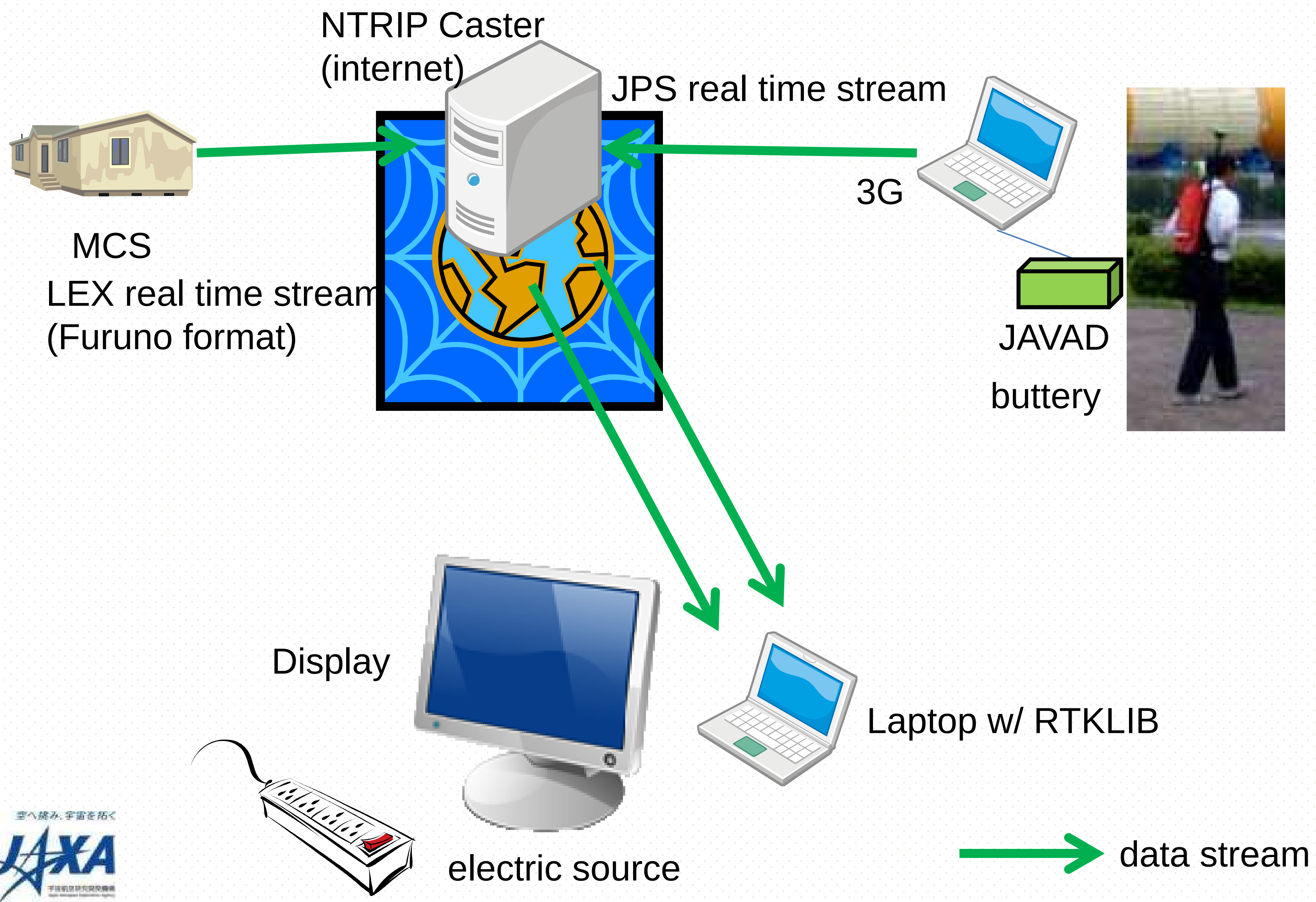


Antenna
LEX Rx
LEX real time stream
(Furuno format)
Wifi Router
（MZK-RP150N）
JAVAD
buttery
JPS real time stream
Display
Laptop w/ RTKLIB
electric source
data stream

System Configuration



rainy day


NTRIP Caster
(internet)
JPS real time stream
3G
MCS
LEX real time stream
(Furuno format)
JAVAD
buttery
Display
Laptop w/ RTKLIB
electric source
data stream